C*- AND W*-ALGEBRAS OF OBSERVABLES, THEIR INTERPRETATIONS, AND THE PROBLEM OF MEASUREMENT

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1. Introduction

Algebraic formulations of quantum theory are an outstanding example of attempts to axiomatize a theoretical framework after the phase of its discovery. The main pioneering phase in the development of quantum mechanics was finished in the early 1930s and has been summarized excellently in von Neumann's monograph (1932) on the mathematical foundations of quantum mechanics. Algebraic quantum theory started in the mid 1930s essentially with Jordan's approach toward non-associative algebras of observables, so-called Jordan algebras. Mathematicians and mathematical physicists like von Neumann, Gel'fand, Naimark, Segal, Wightman, Emch, Haag, and many others have contributed to the further development of algebraic quantum theory. A compact account of the main features of this development has been given by Primas (1983).

The main conceptual difference between pioneer quantum mechanics and algebraic quantum mechanics consists in their different points of departure. The pioneers of quantum mechanics started with classical ideas and tried to find how these correspond to the typical features of quantum systems. Algebraic quantum theory starts with the most typical one of these features, the non-commutativity of quantum mechanical observables, and is basically characterized as a representation theory of the corresponding canonical commutation relations.

This different perspective allows classical systems to be represented within algebraic quantum theory. Methodologically, this representation does not make use of any quantization procedures of classical systems and it does not require any correspondence principle as in the understanding of Bohr. Algebraic quantum theory tries to derive (commutative) classical observables from a basic (non-commutative) quantum structure. In this sense, a basic idea behind algebraic quantum theory is the conviction that quantum theory is a fundamental description of the physical world.

The modern achievements of algebraic quantum theory make clear in what sense pioneer quantum mechanics as well as classical mechanics can be considered as limiting cases of the general theory (Primas 1990a). Compared to the framework of von Neumann's monograph (1932), important extensions are obtained by giving up the irreducibility of the algebra of observables (admitting only non-commuting observables) and the restriction to locally compact state spaces (admitting only finitely many degrees of freedom). As a consequence, modern quantum physics is able to deal with open systems in addition to isolated ones, it can involve infinitely many degrees of freedom such as the modes of a radiation field, it can properly consider interactions with the environment of a system, superselection rules can be formulated that would be impossible in an irreducible algebra of observables, there are in general infinitely many representations inequivalent to the Schrödinger representation, and irreversible (non-unitary) dynamical evolutions can be successfully incorporated.

In addition to this remarkable progress, it has become possible to address quite a number of unresolved conceptual and interpretational problems of pioneer quantum mechanics from a new perspective. This has been a major focus of the work of Hans Primas during the last two decades. First of all, his contributions to a clarification of a number of different concepts of states as well as observables provide a much better understanding of many confusing issues in earlier conceptions. Second, a clear-cut characterization of these concepts is a necessary precondition to explore new approaches, beyond von Neumann's projection postulate, toward the central problem that pervades all quantum theory since its very beginning: the measurement problem.

In our present contribution we intend to describe in detail the progress which has been achieved concerning three different categories of states, observables, and their associated dynamics together with their relevant interpretation. This categorization makes essential use of the distinction between C*- and W*-algebraic formulations; in this respect it is an original contribution even within algebraic quantum theory. Subsequently we shall discuss the measurement problem, basically in two steps: (i) the Heisenberg cut that serves as a metaphor for a proper dressing procedure, i.e., a proper tensorization of the pure state of an isolated system into pure states forming a product state, and (ii) the actual process of measurement, i.e., the dynamics of the product state (object plus environment) that can finally lead to the "material constitution" of an object in an operationally accessible sense. A number of indications will be given for the way in which the concept of a non-classical time operator (in several differing formulations) plays a major role for a proper dynamical description of measurement.

2. States and Observables

2.1. ONTIC STATES WITH INTRINSIC OBSERVABLES

A *strictly isolated* quantum system is in a *pure* state ϕ_t for any time t and evolves under unitary Schrödinger dynamics. The pure state ϕ is called an *ontic* state. It provides a complete specification of the system and is independent of any measurement- or observation-based information gained by external observers. For such a system, concepts like entanglement with its environment and dissipation of energy do not make sense. A pure state ϕ is often represented by a state vector Ψ in the sense that the expectation value $\phi(\hat{A})$ of an observable $\hat{A}$ is given by $\phi(\hat{A}) = \langle\Psi|\hat{A}\Psi\rangle$.

The observables of a strictly isolated quantum system generate a C*-algebra $\mathcal{A}$ (Primas 1994) and are called *intrinsic* observables. They are defined without any reference to empirical measurement or observation. Moreover, isolated quantum systems as such are not observable since any observation would destroy the property of being isolated. They are sometimes called *endo-systems* (Primas 1994), expressing the fact that they do not refer to any external perspective. They are precisely the way they are, and any limited knowledge or partial information of observers is irrelevant at this level of consideration.

From a conceptually rigorous point of view, traditional quantum theory in the framework of the Stone-von Neumann theorem (sometimes called "pioneer quantum mechanics") deals exclusively with ontic states since it deals exclusively with isolated systems. In this framework, the relevant algebra $\mathcal{A}$ of observables is always considered to be irreducible (von Neumann's irreducibility postulate; no commuting observables), and every representation of the canonical commutation rules is considered to be equivalent to a Schrödinger representation.

A strictly isolated quantum system described by a particular pure state ϕ has a *holistic* structure which is encoded in ϕ. In general, it is not legitimate to consider an endo-system to be constituted by sub-endosystems, since the restrictions of the pure state ϕ to the subsystems in general are not pure states any more. Therefore a system in its pure state is an *individual, single* quantum system which does not consist of subsystems (e.g., objects). Since such a system is *not* observable without breaking the isolation from its environment, there is *no* "reduction of the wave function"; hence all types of stochastic dynamics (leading to different eigenstates of any measured observable, with different probabilities) do not play any role. Similarly, transition probabilities between different pure states (as, e.g., specified by Fermi's Golden Rule) do *not* make sense.

2.2. EPISTEMIC STATES WITH CONTEXTUAL OBSERVABLES

For any *open* quantum system, sometimes called *exo-system*, the coupling to its environment is relevant and cannot be neglected. Even if the joint quantum system {exo-system & environment} is isolated and hence in a pure state ϕ, the restriction of ϕ to the exo-system is usually *not* pure any more. Such non-pure states η can be represented by a density operator D, in the sense that for observables $\hat{A}$ of the exo-system the expectation value with respect to the overall pure state ϕ is given as $\phi(\hat{A}) = \eta(\hat{A}) = \mathrm{Tr}(D\hat{A})$, where "Tr" denotes the trace of an operator (the state ϕ refers to all observables, the state η refers to the "relevant" observables of the exo-system only).

The density operator D represents only partial information and does not allow the overall pure state ϕ to be reconstructed. The non-pure state represented by the density operator D is called an *epistemic* state, since it refers to our *knowledge* about the state ϕ. The dynamics of an *open* quantum exo-system is not determined by the Schrödinger equation alone, additional "dissipative" effects usually play a role (Fick and Sauermann 1986, Kubo et al. 1985).

Which structure is appropriate for the observables of a quantum exo-system? Since the joint quantum system {exo-system & rest of the world} is an endo-system and its observables are described by a C*-algebra $\mathcal{A}$, the "relevant" observables of the exo-system generate a C*-subalgebra $\mathcal{B}$ of $\mathcal{A}$. For a given non-pure epistemic state η of the quantum exo-system, one can always construct an appropriate W*-algebra $\mathcal{M}_\eta$, namely the σ-weak closure of the C*-algebra $\pi_\eta(\mathcal{B})$ represented in the Gel'fand–Naimark–Segal Hilbert-space representation (briefly: GNS-representation) π_η of $\mathcal{B}$ with respect to the state η (Bratteli and Robinson 1987, Pedersen 1979, Dixmier 1977). This representation depends always on a context (e.g., a reference state), therefore the corresponding observables are called *contextual* observables (Primas 1994).

The advantage of this procedure is basically that the non-pure state η can now be represented by a density operator (maybe even a vector, also for non-pure states) in the corresponding GNS-Hilbert space. In an abstract C*-algebraic setting, density operators *cannot* be introduced (but non-pure states are well-defined). A further advantage of the W*-algebra is that it contains important classical observables such as operators for temperature (Takesaki 1970) and chemical potential (Müller-Herold 1980), which are not explicitly recognizable in the C*-algebraic formalism. It is often appropriate to construct a W*-algebra on a Hilbert space in such a way that *all* relevant non-pure states of the exo-system can be represented by density operators. This is a non-trivial task if the number of such states is uncountable (and if the Hilbert space should be separable nevertheless).

An exo-system in its non-pure epistemic state η may

- either describe an individual single system, if the state η arises by restriction of a pure state on a larger quantum object,
- or describe an ensemble of individual quantum systems, if the state η is actually a "mixture" of different pure states of the exo-system itself,
- or describe a single individual system, if the state η arises as the *average in time* of pure states ρ_t (though the actual average is not known).

Usually, these different possibilities are not clearly distinguished, because pure states are not considered in an appropriate exo-description.

For an open exo-system described by a particular non-pure state η, consideration of subsystems is admitted, and restriction of the states η_t to some subsystem produces other non-pure states. *Detailed* information about holistic entanglement is *not* encoded in the epistemic states η_t, since it is not clear whether these states arise as restrictions of overall pure states ϕ on a larger system or as averages of pure states of the exo-system itself. Even in the latter case holistic entanglement with respect to a tensorization $\mathcal{B} = \mathcal{B}_1 \otimes \mathcal{B}_2$ is not defined.

2.3. EPISTEMIC STATES WITH INTRINSIC OBSERVABLES

In this section we introduce a third, "hybrid" level of description in addition to those of ontic states with intrinisic observables and epistemic states with contextual observables. It is designed to address situations in which an exo-system is always in a *pure* state. Such an exo-system will be called an *exo-object*. The creation of an exo-object relies upon an appropriate *dressing procedure* (see Sect. 3) which guarantees that the restrictions of all relevant pure states on the overall joint quantum system {exo-system & environment} are pure states again. The dynamics of a quantum exo-object are not determined by the Schrödinger equation alone; additional *stochastic* elements come in which roughly correspond to the dissipative part of the density-operator dynamics for exo-systems as in Sect. 2.2 (see Primas 1990b). For an exo-object, single pure states and *probability distributions* μ *of pure states* are admitted. Averaging over a probability distribution μ of pure states ρ *of the exo-object* results in a *uniquely* determined non-pure state η_μ with density operator D_μ defined by

$$\eta_\mu(\hat{A}) = \mathrm{Tr}(D_\mu \hat{A}) = \int_{\text{all pure states } \rho} \rho(\hat{A})\mu(d\rho). \tag{1}$$

The converse is not true, i.e., to a given non-pure state η represented by a density operator D there exist many different probability distributions μ on the pure states of the exo-object such that Eq. (1) holds. Probability distributions of pure states on an exo-object are called *epistemic probability distribution* or simply *epistemic state* (Amann and Primas 1997), because they refer to our restricted knowledge concerning the "correct" pure state of the exo-object (see also Amann 1995, 1997).

While an *ontic* state refers always to an endo-system, an *epistemic* state may either be a non-pure state (for an exo-system) or a probability distribution of pure states (for an exo-object). Let us stress again that the two different types of epistemic states are not in one-to-one correspondence. To every epistemic state of a quantum exo-object there corresponds uniquely an epistemic state of a quantum exo-system, but the converse is not true.

Which structure is appropriate for the observables of an exo-object? It is the C*-algebra $\mathcal{B}$, without the σ-weak closure to be introduced for the W*-algebra of observables for exo-systems. Surprisingly, a W*-algebra often does *not* make sense for exo-objects! The reason for this is the following: Let us take a probability distribution μ of pure states on the C*-algebra $\mathcal{B}$ with associated non-pure state η (through averaging as in Eq. (1)). This non-pure state η can be *uniquely* extended to a σ-weakly continuous state on the W*-algebra $\mathcal{M}_\eta$ (generated by the C*-algebra $\mathcal{B}$ in the GNS-representation π_η with respect to η; a state on a W*-algebra is σ-weakly continuous if and only if it can be represented by a density operator). Nevertheless the pure states on $\mathcal{B}$ *cannot* be extended to σ-weakly continuous states on $\mathcal{M}_\eta$, and hence also the probability distribution μ (i.e., the epistemic state of the exo-object) does not make sense at the level of the W*-algebra $\mathcal{M}_\eta$. Even worse: Pure states on a typical W*-algebra of type III arising in thermal situations (Bratteli and Robinson 1981) are utterly pathological, because they do not even fulfill the (almost trivial) Jauch-Piron condition (Amann 1987). The reason for this is that the W*-algebra does not only contain interesting observables such as temperature or chemical potential, but also a lot of "garbage" which enters by taking the σ-weak closure of the C*-algebra $\mathcal{B}$. A remarkable exception are factor W*-algebras of type I, i.e., W*-algebras isomorphic to all bounded operators on a Hilbert space $\mathcal{H}$.

These are the algebras of traditional quantum theory, where pure states have particularly nice properties, and fulfill the Jauch-Piron condition.

An exo-object is described by probability distributions μ_t of pure states which – by averaging as in Eq. (1) – provide non-pure states η_t. This depends, of course, on the initial pure state or on the initial distribution $\mu_\circ$ of pure states. For a large class of stochastic dynamics of exo-objects, however, one ends up with *one* particular distribution μ_∞ of pure states in the limit $t \to \infty$ *independently of the initial conditions* (such dynamical exo-objects are called ergodic). Splitting of the underlying C*-algebra $\mathcal{B}$ into two subsystems with two C*-subalgebras $\mathcal{B}_1$ and $\mathcal{B}_2$, $\mathcal{B} = \mathcal{B}_1 \otimes \mathcal{B}_2$, is then admitted under certain conditions. In an ideal situation all those pure states onto which the probability measures μ_t extend are *product* states with respect to the tensor product $\mathcal{B} = \mathcal{B}_1 \otimes \mathcal{B}_2$. This situation never arises in practice, but "most" relevant pure states can be product states or almost product states, if the tensorization is chosen appropriately (see "dressing procedures" in the next section). One can introduce a quantitative criterion estimating a tensorization of $\mathcal{B}$ at time t by the positive number

$$I_{\mathcal{B}_1 \otimes \mathcal{B}_2} \stackrel{\text{def}}{=} \int_{\text{all pure states } \rho} p(\rho)\mu_\infty(d\rho)\,, \tag{2}$$

where $p(\rho)$ estimates the "degree" to which the pure state ρ is a product state (taking, e.g., the value $p(\rho) = 0$, iff ρ is a product state). Let us stress that one must *always accept approximations* when dealing with exo-objects. The exo-object itself may arise by a tensorization of some larger system in the way just described. Consequently one must always expect that the pure states used in the description of an exo-object are not exactly product states.

Remark: Consider the tensor product $\mathcal{B} = \mathcal{B}_1 \otimes \mathcal{B}_2$. Assume that a given probability measure μ_1 exists on product states $\rho_1 \otimes \rho_2$ only, and introduce the respective averaged non-pure state η, as defined in Eq. (1). Then the state η can be decomposed into pure states which are *not* of product form, i.e., there exists another probability distribution μ_2 of pure states on $\mathcal{B}$, which exists on non-product states but nevertheless results in η by averaging. Hence a non-pure state η does not encode the holistic entanglement between two subsystems with C*-algebras $\mathcal{B}_1$ and $\mathcal{B}_2$. For exo-objects one can find appropriate quantitative criteria to estimate the holistic entanglement for a given tensorization (which is impossible if only a non-pure state η is known).

An exo-object is always in a *pure* state, though this pure state is perhaps not precisely known and must be estimated (a probability distribution μ of pure states can be such an estimate). Therefore an exo-object is always an *individual* quantum object, described in a statistical way (namely by an epistemic probability distribution). The stochastic aspect of the time evolution (of pure states of the exo-object) stems from the fact that the (initial) state of the environment cannot be determined (and is therefore a stochastic variable). Starting from an initial pure state $\rho_\circ$, one gets time-evolved states $\rho_{t,\omega}$, where ω is the stochastic variable (i.e., the pure initial state of the environment). In an ergodic situation, the final distribution η_∞ of pure states (for large or infinite time) could – in principle – be regarded as an average in time *or* as a mixture of all the different pure states with respect to the stochastic variable ω. Since we consider an exo-object to be an individual quantum object (and not an ensemble of quantum objects), we shall interpret η_∞ as *average in time* of the pure states $\rho_{t,\omega}$, where the external (initial) state ω is kept fixed.

For a quantum-mechanical description of (mesoscopic or macroscopic) "common sense" objects, an exo-object formalism is appropriate. A cat, for example, is surely not an endo-system and obviously considered as an individual, single object. Hence a cat – when described quantum mechanically – is in a pure state, but this pure state is unstable under small external perturbations and can therefore only be estimated. As stressed above, *approximations are unavoidable* when exo-objects are discussed. An exo-object is described by a pure state only approximately. De facto, entanglement between an exo-object and its environment cannot be excluded completely since both have quantum aspects and since even an optimal dressing is not exact (see Sect. 3).

2.4. SUMMARY

According to the preceding detailed discussion, we have three categories of quantum systems: isolated endo-systems with pure states, open exo-systems with non-pure states, and exo-objects that are open systems with pure states. From a conceptual point of view, the pure states of endo-systems are ontic in the sense that they are independent of (epistemic) information or knowledge that observers may have about them. By contrast, the states of exo-systems and exo-objects are obviously epistemic, however in different ways. While states of exo-systems are described by density operators, states of exo-objects are described by probability distributions of pure states.

The intrinsic, operationally inaccessible observables of an endo-system are elements of a C*-algebra $\mathcal{A}$. If an endo-system is decomposed into an exo-object and its environment (by an appropriate dressing procedure), the C*-algebra of observables of this exo-object is given by a "relevant" subset $\mathcal{B} \subset \mathcal{A}$. The algebra of contextual observables of exo-systems is a W*-algebra $\mathcal{M}$, obtained by the σ-weak closure of $\mathcal{B}$ which can be obtained by a (contextual) GNS-construction. The corresponding three categories of dynamics are a Schrödinger dynamics of pure states for an endo-system, density-operator dynamics including Schrödinger dynamics and dissipative terms for exo-systems, and Schrödinger dynamics of pure states and stochastic terms for exo-objects.

These three categories of quantum systems need different interpretations. An individual and non-statistical interpretation is appropriate for endo-systems, a "density-operator statistical" interpretation is required for exo-systems, and an individual-statistical interpretation for exo-objects. There are two different interpretations for the different epistemic state concepts, expressing the notorious difference between a probability distribution (of pure states) as such and its moments (averages).

3. Heisenberg Cut and Dressing

By contrast to common-sense guided expectations, a "material object" is not at all defined in an obvious, a priori manner. Consider, for example, something like a molecule as such a material entity with its common molecular Hamiltonian

$$\hat{H} \overset{\mathrm{def}}{=} \sum_j \frac{\hat{P}_j^2}{2m_j} + \frac{1}{4\pi\epsilon_\circ} \sum_{j\,<\,k}\sum \frac{e_j e_k}{|\hat{\vec{q}}_j - \hat{\vec{q}}_k|}\,, \tag{3}$$

where the sums go over all the particles (electrons and nuclei) of the molecule, m_j are the respective masses and e_j are the respective charges. Such a Hamiltonian describes already a *dressed object*. "Originally" only the joint system {molecule & radiation field} is well-defined, since charged particles cannot be screened from the radiation field. The Hamiltonian of the joint system consists of three parts comprising, firstly, the kinetic energy of the nuclei and electrons, secondly, the energy of the radiation field and, thirdly, the interaction energy between the charged particles and the radiation field. By a clever choice of conventions one can single out from the third part an interaction between the charged particles, which is – by convention – *attributed* (Heitler 1954) to the molecule and appears now as the potential energy in Eq. (3). It could, of course, also be attributed to the radiation field, since it is the field which mediates the interaction. This choice/attribution corresponds to an appropriate "cut" of the joint endo-system

{molecule & radiation field},

described by a pure state ϕ, into a product of pure states according to

{dressed molecule & dressed radiation field}.

The lesson is that molecules as objects are not defined a priori. A clever choice is necessary and the resulting object is called a "dressed quantum object", e.g., a dressed molecule. The idea behind all this is that such a dressed object is in a pure (approximately pure) quantum state ρ_{mol}. Consequently, the joint system {dressed molecule & dressed radiation field} is a product state $\rho_{\mathrm{mol}} \otimes \rho_{\mathrm{rad}}$ of two pure states.

It is not clear from the very beginning whether such conventions are sensible. A choice or cut as mentioned above does not necessarily lead to a quantum *object*, but could result in a quantum *system* only (both exo, of course). In the latter situation, pure states would usually not play a role and should be replaced by non-pure states (density operators). In the particular case of molecules, this would mean that neither molecular structure, nor handedness, nor the monomer sequence of a macromolecule would make sense (Amann 1995, 1996, 1997), because non-pure states (such as thermal states) can be decomposed into pure states that may or may not admit molecular structure (handedness etc.).

Similarly, considering a cat from a quantum-mechanical point of view, we would hope that it is a quantum exo-object and not just a quantum exo-system. Being a quantum exo-system only would imply that holistic features (between different cats or between a cat and its environment) play a decisive role. Being a quantum exo-object means that only "internal" holistic effects persist rather than holistic entanglement with the environment. In the terminology of Sect. 2, molecules, cats, etc. are to be considered as exo-*objects*. Hence they are – as a rule – described by pure states, though non-pure states may play a certain role, e. g., for the spontaneous decay of molecular states.

Dressing procedures go by two steps:

(a) First one tries to reformulate the Hamiltonian, suggesting new particles or other structures. Second quantization (Guichardet 1970, Jost 1973), for example, is a procedure which starts with a direct-sum Hilbert space composed of subspaces referring to a fixed number n of "old" particles (arbitrary $n = 1, 2, \ldots$); then boson (or fermion) operators are introduced which give rise to "new particles", e.g., photons or phonons (whose observables – isomorphic to a factor W*-algebra of type I – are generated by a pair a_j and a_j^* of boson operators satisfying the canonical commutation relation $[a_j, a_m^*] = \delta_{jm}\mathbb{1}$, with commuting operators for different new particles). Nevertheless, non-product states between different "new particles" are admissible, i.e., the "new particles" are not necessarily in pure states (not necessarily exo-objects, but maybe only exo-systems).

(b) Secondly, one might insist that "new particles" are (approximate) exo-*objects*, either by declaration or by comparison with experiment or by some theoretical derivation. Often, this is indeed a mere *declaration*, which "forbids" certain superpositions of given basic pure states.

Usually, experiments describe ensembles of quantum systems/objects and therefore depend only on the averaged density-operator dynamics and not on the actual stochastic dynamics of the pure states. Such experiments cannot distinguish between different declarations (as long as both declarations are compatible with the density-operator dynamics). Consequently, such declarations are of practical use only and do not reproduce the reality of individual quantum objects.

Here we are mainly interested in a description of individual, possibly macroscopic quantum objects. Experiments then do *not* refer to an ensemble of such objects, but to individual systems or objects. In such a situation the distinction between exo-systems and exo-objects is vital and new structures cannot be introduced by mere declarations.

How can a proper dressing procedure be introduced in such an "individual" context? Let us stress again the dichotomy between dressed exo-objects on the one hand and quantum mechanical superpositions on the other:

- For proper dressed objects it is vital to have (approximate) pure product states $\rho_1 \otimes \rho_2$ for the joint system {dressed object & dressed environment}.
- The superposition principle is at the heart of quantum mechanics, but superpositions of product states are *not* product states any more. Their restriction to the dressed object or dressed environment is therefore a non-pure state.

An appealing way out of this conflict is a *stability analysis* of the pure states on the joint system {dressed object & dressed environment}. We consider this joint system to be described by a C*-algebra $\mathcal{B}$ from which the dressed exo-object is derived, with pure states under a stochastic dynamics. Though *all* pure states of this joint system "exist", some of them are unstable with respect to the stochastic dynamics and decay into more stable ones. As mentioned in Sect. 2.3, the final distribution μ_∞ of pure states for $t \to \infty$ is independent from the initial conditions if the dynamics is ergodic. Then different tensorizations $\mathcal{B} = \mathcal{B}_1 \otimes \mathcal{B}_2$ can be estimated quantitatively by the positive number $I_{\mathcal{B}_1 \otimes \mathcal{B}_2}$ defined in Eq. (2). If $I_{\mathcal{B}_1 \otimes \mathcal{B}_2} = 0$, then all relevant pure states are exact product states with respect to the chosen tensorization.

In all practical cases, one must expect that $I_{\mathcal{B}_1 \otimes \mathcal{B}_2}$ takes small but non-zero values and non-product states play some role. Even if $I_{\mathcal{B}_1 \otimes \mathcal{B}_2}$ is equal to zero, it is only guaranteed that the final distribution μ_∞ exists on product states. It is not guaranteed that an initial product state will be a product state for all finite times $t > 0$. Hence one may start with a product state and end with a product state (for large times), and nevertheless intermediate states can be of non-product form.

Usually, such non-product states are suppressed – either by accepting a Hartree approximation (Primas 1990b,c) from the very beginning (only product states are admitted) or by considering an appropriate *limit* with respect to which only product states appear (e.g., infinite volume, infinitely many particles, infinite nuclear masses in molecules, etc.). In limit situations, classical structures and classical observables may appear "automatically" through the mathematical formalism (Müller-Herold 1980, Bratteli and Robinson 1981, Primas 1983, Sewell 1986), partly suppressing the consequences of the superposition principle. In realistic situations one has, of course, finite volume, finitely many particles, or finite nuclear molecular masses, and so on, with unrestricted validity of the superposition principle. Hence, one is in the strange situation that for arbitrarily large but finite volume, finite nuclear masses, ..., the superposition principle holds unrestrictedly whereas in the limit of infinite volume, ..., its validity is lost. (More precisely: its validity is lost for classical observables in different superselection sectors, separated by superselection rules.)

From our point of view it is important to accept non-product states (with corresponding holistic entanglement between $\mathcal{B}_1$ and $\mathcal{B}_2$) and to quantitatively determine the probability that they appear. This means that

- we accept the Hartree approximation only as a first step toward a proper understanding of dressing procedures and non-linear stochastic time evolutions,
- and we are not only interested in limit considerations, but also in the discussion of systems/objects before the limit is reached, i.e., for large but finite volume etc.

In some relatively simple situations it is possible to discuss these problems explicitly: For a magnet consisting of N spins (each described by Pauli matrices $\sigma_x,\ \sigma_y,\ \sigma_z$) one may investigate the specific magnetization operator

$$\hat{m}_N \stackrel{\text{def}}{=} \frac{1}{N} \sum_{j=1}^{N} \sigma_{z,j} \ . \tag{4}$$

In the limit $N \to \infty$ one gets a classical magnetization $\hat{m} \stackrel{\text{def}}{=} \lim_{N\to\infty} \hat{m}_N$, i.e., $\hat{m}$ commutes with *all* other observables (constructed from local Pauli matrices). This fact implies that – in the limit – pure states with different expectation values of $\hat{m}$ are not superposed any more. It is no problem to verify these facts mathematically, but they are not easily understandable from a conceptual point of view, since such superpositions can be written down as pure states for any (whatever large but) *finite* number N of spins. A stability analysis in the case of a Curie–Weiss magnet (Amann 1995, Amann and Primas 1997) shows that these superpositions "die out" successively with increasing number N of spins, and the exponential rate of this process is given by an entropy in the sense of large deviations statistics.

4. Dynamics of Measurement

Any dynamical description of measurement has to start from a proper decomposition of a system into a dressed exo-object and its dressed environment. It is crucial to keep in mind that such a decomposition is a logical precondition for the dynamics of measurement insofar as the Hamiltonian of the composed system needs to be written as a sum

$$H = H_{\rm obj} \otimes 1 + 1 \otimes H_{\rm env} + H_{\rm int}. \tag{5}$$

An illustrative example has been extensively discussed by Primas (1990b,c). Consider the simple case of a two-level quantum object (spin 1/2 system) with the Hamiltonian

$$H_{\rm obj} = \frac{\hbar}{2} \sum_{\nu=1}^{3} \Omega_\nu \sigma_\nu, \tag{6}$$

a sufficiently nontrivial boson field environment

$$H_{\rm env} = \sum_{\nu=1}^{3} \sum_{k} \omega_k \alpha_{k\nu}^* \alpha_{k\nu}, \tag{7}$$

and an interaction

$$H_{\rm int} = \sum_{\nu=1}^{3} \sigma_\nu \otimes A_\nu\,, \tag{8}$$

where

$$A_\nu = \sum_k \lambda_{k\nu} \alpha_{k\nu} + c.c. \tag{9}$$

If such a decomposition has been properly carried out (cf. Sec. 3), then it is possible to derive the expectation values

$$\begin{aligned} M(t) &= <\psi_t|\sigma|\psi_t> && (10)\\ \alpha(t) &= <\chi_t|A|\chi_t> && (11) \end{aligned}$$

with respect to the (approximate) product state

$$\Psi_t = \psi_t^{\rm obj} \otimes \chi_t^{\rm env}. \tag{12}$$

Corresponding to the product state Ψ_t, the C*-algebra of intrinsic observables in the composed system of dressed object and dressed environment is

$$\mathcal{A} = \mathcal{A}_{\rm obj} \otimes \mathcal{A}_{\rm env}. \tag{13}$$

$\mathcal{A}_{\mathrm{obj}}$ is the C*-algebra of 2×2 matrices and $\mathcal{A}_{\mathrm{env}}$ is the C*-algebra of intrinsic observables of an environment with infinitely many degrees of freedom.

The equations of motion for the expectation values $M(t)$ and $\alpha(t)$ are given by:

$$\dot{M}(t) = M(t) \times \Omega + M(t) \times \alpha(t)\,, \tag{14}$$

$$\dot{\alpha}_{k\nu}(t) = -\omega_k \alpha_{k\nu} + \frac{\mathrm{i}}{2}\lambda_{k\nu} M_\nu(t)\,. \tag{15}$$

They describe the feedback between object and environment. More precisely, they describe the polarization M of the object under the influence of the environment and the motion of the environment observable α (boson operator) under the polarizing influence of the object. The solution of the second equation, referring to the observables of the environment (or the measuring system, respectively) splits into a retarded and an advanced part:

$$\begin{aligned}\alpha_{k\nu}^{\mathrm{ret}} &= \exp(-i\omega_k t)\alpha_{k\nu}(0) \\ &- \frac{\mathrm{i}}{2}\lambda_{k\nu}\int_0^t \exp(-i\omega_k(t-s))M_\nu(s)ds \quad (t\geq 0)\,,\end{aligned} \tag{16}$$

$$\begin{aligned}\alpha_{k\nu}^{\mathrm{adv}} &= \exp(-i\omega_k t)\alpha_{k\nu}(t) \\ &+ \frac{\mathrm{i}}{2}\lambda_{k\nu}\int_t^0 \exp(-i\omega_k(t-s))M_\nu(s)ds \quad (t\leq 0)\,,\end{aligned} \tag{17}$$

Selecting one of these solutions and disregarding the other requires the time inversion symmetry of the composed system to be broken. In other words: a bidirectionally deterministic system has to be described in terms of a superposition of a backward deterministic (forward non-deterministic) and a forward deterministic (backward non-deterministic) process which are equally relevant a priori. Then, one can apply the principle of causality (past-determinacy, error-free retrodiction, no anticipation) as a "heuristic" argument for the selection of one of the solutions. Causality is consistent with the retarded solution, whereas the advanced solution contradicts causality, i.e., it has the properties of future-determinacy, error-free prediction, and no memory (Primas 1992).

It has been argued that the retarded, i.e., the backward deterministic, forward non-deterministic, solution is a K-flow on a state space with infinitely many degrees of freedom (Primas 1990b,c, 1997a). Mixing alone is *not* sufficient to break the time reversal symmetry of the interaction between object and environment (Primas 1997b). In the simplest case, the relaxation time for this K-flow is the time constant τ_ν of an exponentially decaying correlation function (for details, see Primas 1990b)

$$K_\nu = \gamma_\nu \exp(-|t|/\tau_\nu). \tag{18}$$

At this point we are still at the level of description of intrinsic observables, needed for the specification of initial conditions of the K-flow. Conceptually, this K-flow represents a stochastic process for the exo-object (cf. Pearle 1976, Gisin 1984, Ghirardi et al. 1986) which corresponds to chaos in the sense of Wiener rather than chaos in the sense of Kolmogorov and Sinai (i.e., a dissipative exo-system dynamics). The transition from the former to the latter is made by averaging over a probability distribution of pure states and their trajectories, i.e., by introducing the level of description using density operators or, in classical systems, distribution functions and corresponding ensembles of trajectories. After such a procedure one deals with epistemic states and (GNS-constructed) contextual observables, where a context may have been introduced by a reference state with respect to which stability in a certain sense (hopefully more general than thermal equilibrium) can be checked. At this level of description, the evolution of the flow is an irreversible semigroup evolution with a finite number of degrees of freedom, whereas Wiener chaos is a stochastic process in an infinite-dimensional state space.

The fact that the dynamics of measurement can be described as a stochastic process in time suggests that it takes a certain amount of time until a measurement is finished. Of course, this statement applies to the act of measurement in the general sense of an interaction between an object and its environment, approximately decomposed from one another, and is not restricted to controlled laboratory measurements. The temporal cut determining a measurement as finished "by declaration" is intimately related to the Heisenberg cut approximately separating an object from its environment. It may be speculated that the relaxation time τ_ν of the K-flow is a function of the degree of stability of the pure states forming the dressed product state.

Since these deliberations apply to non-controlled types of "measurement" in a quite general sense, one may make use of them for the notorious discussion of "quantum jumps" or, more general, any relaxation (or excitation) of unstable quantum objects. Such relaxations do not occur for strictly isolated objects; for instance, an environment is needed to embed an atom at least in its own electromagnetic field. In this context, an interesting analog to the issue of the duration of measurements is the question as to whether quantum jumps happen in time (and, if yes: how much time they take). In the following we sketch how this question might be addressed from a novel perspective, providing an alternative to recent decoherence approaches as represented in Giulini et al. (1996).

An important implication of a K-flow is the fact that the corresponding system admits a time observable which is not in the center of the W*-algebra of contextual observables, i.e., does not represent a classical observable. There is a long tradition of approaches toward such a time observable, which is intimately connected with the history of energy-time uncertainty relations (cf. Atmanspacher 1994). A basic objection against a time operator not commuting with a suitable energy operator (the bounded or even discrete Hamiltonian of a system) has been formulated by Pauli (1933). It has recently been shown in detail (Busch et al. 1994) how Pauli's objection can be circumvented if the relevant time observable is defined by a positive operator valued (POV) measure rather than a projection valued (PV) spectral measure. POV measures are concepts for observables that are more general than PV measures. A POV measure is a PV measure, if it is multiplicative, if the corresponding operator is idempotent and not only Hermitian but also self-adjoint, and if the set of its eigenfunctions (if they exist) is orthogonal.

If H is the Hamiltonian of a system S, then $t \mapsto \mathrm{e}^{-\mathrm{i}Ht}$, $t \in \mathbb{R}$, is a unitary representation V_t of the time translation group. If, furthermore, Θ is a time interval during which an event is (with some probability) expected to occur in a certain state of S, then a suitable dynamical variable $B(\Theta)$ satisfies

$$V_t^* B(\Theta) V_t = B(\Theta - t), \qquad V_t = \mathrm{e}^{-\mathrm{i}Ht}. \tag{19}$$

Such a dynamical variable is a POV measure for a time observable (Busch et al. 1994). Its construction is possible in specific cases, but cannot be universally prescribed; it depends crucially on contexts given by the system considered. On the basis of B, a time operator (briefly: POV time) can be defined according to

$$T_B = \int t\, B\,(dt), \tag{20}$$

which is *not* self-adjoint and fulfills the commutation relation

$$\mathrm{i}[H, T_B] = \mathbb{1} \tag{21}$$

on an appropriate domain, without contradicting Pauli's theorem (Busch et al. 1994).

From a different point of view, Misra and others (Misra 1978, Misra et al. 1979a,b; see also Suchanecki 1992) have introduced a time operator T_L on the basis of the Liouville representation of a dynamical system. A related definition of a time operator, based on the theory of stochastic processes, has been proposed by Tjøstheim (1976) and Gustafson and Misra (1976); see also Primas (1997a). For both approaches, the condition of a K-flow is crucial. In the Liouville representation, the time operator is a shift operator T_L

$$U_t^* T_L U_t = T_L + t\mathbb{1}, \qquad U_t = \mathrm{e}^{-\mathrm{i}Lt}. \tag{22}$$

T_L does not commute with the Liouvillean L that can be formulated as the Poisson bracket $L\rho = \{H, \rho\}$,

$$\mathrm{i}[L, T_L] = \mathbb{1}, \tag{23}$$

and hence does not contradict Pauli's theorem either (Misra 1978).

The two time operators are not the same, and their relationship with each other is not immediately clear. An important difference between them is given by the fact that POV time is in general not self-adjoint, whereas a time operator in the sense of Misra is a self-adjoint PV measure (for more details, see Atmanspacher and Amann 1998). Both T_B and T_L are defined as shift operators, but with respect to different generators of the dynamical evolution (H and L, respectively). If the eigenstates of a system are non-stationary, energy levels due to eigenvalues of H have a non-vanishing width ΔE corresponding to the fact that some dynamical variable, as required for the definition of POV time, is *not* a conserved quantity, i.e., its expectation value changes in time, $d\langle B\rangle/dt \neq 0$ (cf. Mandelstam and Tamm 1945).

This bandwidth can be expressed as the eigenvalue of a PV operator L (cf. Prigogine and Petrosky 1987, 1988). The interpretation of Misra's T_L as an observable is different from that of POV time. POV time refers to the "time of occurence" of an event in a non-stationary state as an observable canonically conjugate to a proper Hamiltonian. By contrast, Misra's T_L refers to the "eigentime" of a non-stationary state as an observable canonically conjugate to the Liouvillean considered as an operator whose eigenvalues are differences of energies, e.g., bandwidths ΔE. (Grelland (1993) has shown that those eigenvalues of L vanish if the eigenstates of H are stationary. See also Ban (1991)).

The non-commutative character of a time operator in the Liouville representation can be operationalized if one moves from T_L to an information operator M (Atmanspacher and Scheingraber 1987) according to

$$U_t^* M U_t = M - h_T t\mathbb{1}, \qquad U_t = \mathrm{e}^{-\mathrm{i}Lt}, \tag{24}$$

where h_T is the empirically accessible (Grassberger and Procaccia 1983) Kolmogorov-Sinai (KS) entropy of the system (Kolmogorov 1958, Sinai 1959). It is important to realize that the definition of M is *more general* than that of T_L insofar as M can be defined even for non-mixing ergodic systems with $h_T = 0$, whereas a necessary condition for the existence of T_L is strong mixing, sometimes even $h_T > 0$. While T_L, if it exists, does not commute with the Liouvillean L, M commutes with L iff $h_T = 0$, and M does not commute with L iff $h_T > 0$ (cf. Atmanspacher and Scheingraber 1987, Atmanspacher 1997):

$$\mathrm{i}[L, M] = h_T\mathbb{1}. \tag{25}$$

The KS entropy h_T as the key quantity in this framework can (somewhat roughly) be defined as the sum of positive Ljapunov exponents of the system (for more details, see Atmanspacher 1997). The Ljapunov exponents are basically derived from eigenvalues of a linear stability matrix which refers to a comoving (with the flow) coordinate system. Hence it is intuitively clear that the set of its eigenfunctions is orthogonal only locally in state space (at a given time). (Compare recent work of Grossmann and collaborators (Grossmann 1996) on the problem of non-orthogonal eigenfunctions of non-normal shear operators in turbulent flows.) In addition to other features (Davies and Lewis 1970, Davies 1970, Srinivas 1980, Ozawa 1984, Primas 1990b), this illuminates the use of POV measures, in particular POV time, to characterize K-flows in measurement theory (and otherwise).

An additional interesting feature in this context is the fact that h_T is an inverse relaxation time or, more precisely, an inverse predictability time (see Atmanspacher and Scheingraber 1987, Atmanspacher 1997). At a speculative level, one might think of a quantitative relationship between h_T and $1/\tau_\nu$ (in simple cases). This would allow us to characterize the duration of a measurement by the dynamical invariant (h_T) of a nonlinear process in a finite-dimensional state space (KS chaos) as well as a relaxation time of a linear process in an infinite-dimensional state space (Wiener chaos).

At the same time, it is interesting to think of the properties of KS chaos in terms of an information flow (Goldstein 1981, Shaw 1981, Farmer 1982, Caves 1994) which always accompanies the transition from ontic to epistemic states. The transition from an infinitely refined partition for ontic states to binary alternatives has been addressed by Atmanspacher (1989) for the act in which "virtual particles realize themselves" – another example for a generalized understanding of the process of measurement. It is of interest that K-flows entail an intrinsic and unique, but system-specific (contextual) definition of a so-called generating partition (Cornfeld et al. 1982; see also Crutchfield 1983) which is *not* imposed ad hoc or with the help of additional arguments. For such a generating partition, trajectories which are very close to each other initially need an average time of the order of the inverse of h_T to become sufficiently separated to be distinguishable with respect to the generating partition.

If measurement is generically related to K-flow properties (Lockhart and Misra 1986, Primas 1997a), then a time observable in the sense of Misra exists which does not commute with all other observables, i.e., which does not belong to the center of the algebra of contextual observables of the exo-system considered. At the level of the exo-object, the K-flow properties refer to the possible histories (trajectories) corresponding to the temporal evolution of the probability distribution of pure states. At this level of description it is appealing to apply the concept of consistent histories (Omnès

1992) in order to study temporal Bell inequalities (Leggett and Garg 1985, Paz and Mahler 1993, Mahler 1994). Analogous to the standard Bell inequalities, one can show that their temporal counterparts may violate the classical imagination of consistent histories satisfying probability sum rules. The interpretation of inconsistent histories in this sense is due to a temporal nonlocality which is bound to a system-specific time scale (relaxation time) and expresses itself as a lack of well defined temporal order (before-after relationships) within that time scale. Inconsistent histories are relevant at the level of endo-systems rather than exo-objects or exo-systems.

The precise connections between such a temporal nonlocality, the existence of a non-commuting time observable, and the relaxation properties of measurement processes have yet to be explored in detail. Some indications and perspectives have been sketched in Atmanspacher (1997). An important point is that there is more than one way to look at the process of measurement and corresponding topics such as quantum jumps etc. Possibility (1) is to look at the measurement process in terms of the evolution of expectation values of observables as a function of a parameter time, together with their temporal relaxation properties. A second way (2) is to consider measurement from the perspective of temporal nonlocality, associated with the absence of temporal order within the time interval which (1) provides for the relaxation. This time interval is related to the information flow rate represented by the KS entropy. Such a temporally nonlocal interval corresponds to the existence of a time operator (POV or Misra) outside the center of the relevant algebra of observables. There are other possibilities to define time operators (cf., e.g., Amann 1986, Isham 1993) which are outside the scope of this paper.

Let us finally return to the symmetry breaking in time due to the split into retarded and advanced solutions for the feedback between object and environment. Cramer (1980, 1986) has suggested considering this split in the framework of his transactional interpretation of quantum mechanics. After the completion of an emitter-absorber transaction, the resulting superposition of retarded and advanced components can be reinterpreted due to a purely retarded solution. (For a comprehensive presentation of this and other approaches toward questions about the arrow of time from the viewpoint of the philosophy of physics compare the recommendable book by Price (1996).) Before the transaction is completed, everything is spatially and temporally nonlocal, such that the two solutions are indistinguishable. It is interesting to look at this feature from the perspective of Misra's (1995) result that the Klein-Gordon evolution of massive particles (not of photons!) admits a time operator if conditions are satisfied that are set by the restriction of self-adjointness (cf. the discussion in Atmanspacher 1997). Since the characteristic time scale associated with the resulting time operator is mass-dependent, one might speculate that there are mass-dependent effects in EPR experiments with massive particles. So far, all uncontroversial conclusive evidence for nonlocal correlations in quantum systems has been gained in experiments with photons, although some experiments with massive particles have been carried out (cf. the collection of articles in Selleri (1988), see also new proposals of EPR type experiments with massive particles by Fry et al. (1995) and Freyberger et al. (1996)).

The symmetry between advanced and retarded solutions is unbroken at the level of ontic (pure) states of the holistic isolated endo-system. At present, we do not know in what precise sense their explicit distinction has to do with the Heisenberg cut. There certainly are connections, but their nature in detail is still unclear. A fascinating idea in this context, advocated by Costa de Beauregard (1987), suggests that symmetry breaking with respect to temporal direction goes hand in hand with the Cartesian cut between the material world of res extensa and its mental counterpart. Bringing this idea into contact with the argumentation in the present article leads to the speculation that the scientific concept of causality (the Aristotelian causa efficiens), referring to backward deterministic processes in the material world, could be complemented by the scientifically disregarded concept of finality (the Aristotelian causa finalis), consistent with forward deterministic, "goal-oriented" processes (Primas 1992) in the mental world. The maximum entropy principle that has been suggested (compare Sec. 3) to find most stable decompositions, i.e., most appropriate Heisenberg cuts, for a given endo-system, effectively represents an example of such a final concept. We may wonder whether this indicates any relation to the advanced counterpart of the retarded solution for proper observables in the environment.

References

Amann A. (1986): Observables in W*-algebraic quantum mechanics. *Fortschr. Phys.* **34**, 167–215.

Amann A. (1987): Jauch-Piron states in W*-algebraic quantum mechanics. *J. Math. Phys.* **28**, 2384–2389.

Amann A. (1995): Structure, dynamics and spectroscopy of single molecules: A challenge to quantum mechanics. *J. Math. Chem.* **18**, 247–308.

Amann A. (1996): Can quantum mechanics account for chemical structures? In *Fundamental Principles of Molecular Modeling*, ed. by W. Gans, J. Boeyens, and A. Amann (Plenum, New York), pp. 55–97.

Amann A. (1997): Fuzzy classical structures in genuine quantum systems. In *Fuzzy Logic in Chemistry*, ed. by D. Rouvray (Academic Press, San Diego), pp. 91–138.

Amann A. and Primas H. (1997): What is the referent of a non-pure quantum state? In *Potentiality, Entanglement, and Passion-at-a-Distance. Quantum Mechanical Studies in Honor of Abner Shimony*, ed. by R.S. Cohen, M. Horne, and J. Stachel (Kluwer, Dordrecht), pp. 9–29.

Atmanspacher H. (1989): The aspect of information production in the process of observation. *Found. Phys.* **19**, 553–577.

Atmanspacher H. (1994): Is the ontic/epistemic distinction sufficient to represent quantum systems exhaustively? In *Symposium on the Foundations of Modern Physics 1994*, ed. by K.V. Laurikainen, C. Montonen, and K. Sunnarborg (Editions Frontières, Gif-sur-Yvette), pp. 15–32.

Atmanspacher H. (1997): Dynamical entropy in dynamical systems. In *Time, Temporality, Now*, ed. by H. Atmanspacher and E. Ruhnau (Springer, Berlin), pp. 327–346.

Atmanspacher H. and Amann A. (1998): Positive operator valued measures and projection valued measures of non-commutative time operators. *Int. J. Theor. Phys.* **37**, 629–650.

Atmanspacher H. and Scheingraber H. (1987): A fundamental link between system theory and statistical mechanics. *Found. Phys.* **17**, 939–963.

Ban M. (1991): Relative number state representation and phase operator for physical systems. *J. Math. Phys.* **32**, 3077–3087.

Bratteli O. and Robinson D.W. (1981): *Operator Algebras and Quantum Statistical Mechanics Vol. 2* (Springer, New York).

Bratteli O. and Robinson D.W. (1987): *Operator Algebras and Quantum Statistical Mechanics Vol. 1.* (Springer, New York, 2nd revised edition).

Busch P., Grabowski M., and Lahti P.J. (1994): Time observables in quantum theory. *Phys. Lett. A* **191**, 357–361.

Caves C. (1994): Information, entropy, and chaos. In *Physical Origins of Time Asymmetry*, ed. by J.J. Halliwell, J. Pérez-Mercader, and W.H. Zurek (Cambridge University Press, Cambridge), pp. 47–89.

Cornfeld I.P., Fomin S.V., and Sinai Ya.G. (1982): *Ergodic Theory* (Springer, Berlin), pp. 250–252.

Costa de Beauregard O. (1987): *Time, the Physical Magnitude* (Reidel, Dordrecht).

Cramer J.G. (1980): Generalized absorber theory and the Einstein-Podolsky-Rosen paradox. *Phys. Rev. D* **22**, 362–376.

Cramer J.G. (1986): The transactional interpretation of quantum mechanics. *Rev. Mod. Phys.* **58**, 647–687.

Crutchfield J.P. (1983): *Noisy Chaos.* PhD Thesis, University of California at Santa Cruz, published by University Microfilms Intl., Minnesota, 1983, Sect. 5.2 (pp. 5.9–5.14) and Sect. 5.4 (pp. 5.19–5.26).

Davies E.B. (1970): On the repeated measurement of continuous observables in quantum mechanics. *J. Functional Analysis* **6**, 318–346.

Davies E.B. and Lewis E.T. (1970): An operational approach to quantum probability. *Commun. Math. Phys.* **17**, 239–260.

Dixmier J. (1977): *C*-Algebras* (North-Holland, Amsterdam).

Farmer D. (1982): Information dimension and the probabilistic structure of chaos. *Z. Naturforsch.* **37a**, 1304–1325.

Fick E. and Sauermann G. (1986): *Quantenstatistik dynamischer Prozesse. Band IIa: Antwort- und Relaxationstheorie* (Harri Deutsch, Thun).

Freyberger M., Aravind P.K., Horne M.A., and Shimony A. (1996): Proposed test of Bell's inequalities without a detection loophole by using entangled Rydberg atoms. *Phys. Rev. A* **53**, 1232–1244.

Fry E.S., Walther T., and Li S. (1995): Proposal for a loophole-free test of the Bell inequalities. *Phys. Rev. A* **52**, 4381–4395.

Ghirardi G.C., Rimini A., and Weber T. (1986): Unified dynamics for microscopic and macroscopic systems. *Phys. Rev. D* **34**, 470–491.

Gisin N. (1984): Quantum measurements and stochastic processes. *Phys. Rev. Lett.* **52**, 1657–1660.

Giulini D., Joos E., Kiefer C., Kupsch J., Stamatescu I.-O., and Zeh H.D. (1996): *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin). See particularly pp. 114–134 in the contribution by Joos.

Goldstein S. (1981): Entropy increase in dynamical systems. *Israel J. Math.* **38**, 241–256.

Grassberger P. and Procaccia I. (1983): Estimation of the Kolmogorov entropy from a chaotic signal. *Phys. Rev. A* **28**, 2591–2593.

Grelland H. (1993): Tomita representations of quantum and classical mechanics in a bra/ket formulation. *Int. J. Theor. Phys.* **32**, 905–925.

Grossmann S. (1996): Instability without instability? In *Nonlinear Physics of Complex Systems*, ed. by J. Parisi, S.C. Müller, and W. Zimmermann (Springer, Berlin), pp. 10–22.

Guichardet A. (1970): *Symmetric Hilbert Spaces and Related Topics. Lecture Notes in Mathematics Vol. 261* (Springer, Berlin).

Gustafson K. and Misra B. (1976): Canonical commutation relations of quantum mechanics and stochastic regularity. *Lett. Math. Phys.* **1**, 275–280.

Heitler W. (1954): *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 3rd revised edition).

Isham C.J. (1993): Canonical quantum gravity and the problem of time. In *Integrable Systems, Quantum Groups, and Quantum Field Theories*, ed. by L.A. Ibert and M.A. Rodriguez (Kluwer, Amsterdam), pp. 157–288.

Jost R. (1973): *Quantenmechanik II* (Verlag des Vereins der Mathematiker und Physiker an der ETH Zürich, Zürich).

Kolmogorov A.N. (1958): A new metric invariant of transitive dynamical systems and automorphisms in Lebesgue spaces. *Dokl. Akad. Nauk SSSR* **119**, 861–864.

Kubo R., Toda M., and Hashitsume N. (1985): *Statistical Physics II* (Springer, Berlin).

Leggett A.J. and Garg A. (1985): Quantum mechanics versus macroscopic realism: is the flux there when nobody looks? *Phys. Rev. Lett.* **54**, 857–860.

Lockhart C.M. and Misra B. (1986): Irreversibility and measurement in quantum mechanics. *Physica A* **136**, 47–76; cf. H. Primas, *Math. Rev.* **87k**, 81006 (1987).

Mahler G. (1994): Temporal Bell inequalities: a journey to the limits of "consistent histories". In *Inside Versus Outside*, ed. by H. Atmanspacher and G.J. Dalenoort (Springer, Berlin), pp. 196–205.

Mandelstam L.I. and Tamm I. (1945): The uncertainty relation between energy and time in non-relativistic quantum mechanics. *J. Phys. (USSR)* **9**, 249–254.

Misra B. (1978): Nonequilibrium entropy, Lyapounov variables, and ergodic properties of classical systems. *Proc. Natl. Acad. Sci. USA* **75**, 1627–1631.

Misra B. (1995): From time operator to chronons. *Found. Phys.* **25**, 1087–1104.

Misra B., Prigogine I., and Courbage M. (1979a): From deterministic dynamics to probabilistic descriptions. *Physica A* **98**, 1–26; cf. A.S. Wightman, *Math. Rev.* **82e**, 85066 (1982).

Misra B., Prigogine I., and Courbage M. (1979b): Lyapounov variable: entropy and measurement in quantum mechanics. *Proc. Natl. Acad. Sci. USA* **76**, 4768–4772.

Müller-Herold U. (1980): Disjointness of β-KMS states with different chemical potential. *Lett. Math. Phys.* **4**, 45–48.

Neumann J. von (1932): *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin). English: *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton), 1955.

Omnès R. (1992): Consistent interpretations of quantum mechanics. *Rev. Mod. Phys.* **64**, 339–382.

Ozawa M. (1984): Quantum measuring processes of continuous observables. *J. Math. Phys.* **25**, 79–87.

Pauli W. (1933): Die allgemeinen Prinzipien der Wellenmechanik. *Handbuch der Physik, Vol. 24*, ed. by H. Geiger und K. Scheel, (Springer, Berlin), pp. 83–272, here: p. 140. Reprinted in S. Flügge, ed. (1958): *Encyclopedia of Physics, Vol. V, Part 1* (Springer, Berlin), pp. 1–168, here: p. 60.

Paz J.P. and Mahler G. (1993): Proposed test for temporal Bell inequalities. *Phys. Rev. Lett.* **71**, 3235–3239.

Pearle P. (1976): Reduction of the state vector by a nonlinear Schrödinger equation. *Phys. Rev. D* **13**, 857–868.

Pedersen G.K. (1979): *C*-Algebras and Their Automorphism Groups* (Academic Press, London).

Price H. (1996): *Time's Arrow and Archimedes' Point* (Oxford University Press, Oxford).

Prigogine I. and Petrosky T.Y. (1987): Intrinsic irreversibility in quantum theory. *Physica A* **147**, 33–47.

Prigogine I. and Petrosky T.Y. (1988): An alternative to quantum theory. *Physica A* **147**, 461–486.

Primas H. (1983): *Chemistry, Quantum Mechanics, and Reductionism. Perspectives in Theoretical Chemistry* (Springer, Berlin).

Primas H. (1990a): Mathematical and philosophical questions in the theory of open and macroscopic quantum systems. In *Sixty-Two Years of Uncertainty*, ed. by A.I. Miller (Plenum, New York), pp. 233–257.

Primas H. (1990b): The measurement process in the individual interpretation of quantum mechanics. In *Quantum Theory Without Reduction*, ed. by M. Cini and J.M. Lévy-Leblond (Adam Hilger, Bristol), pp. 49–68.

Primas H. (1990c): Induced nonlinear time evolution of open quantum objects. In *Sixty-Two Years of Uncertainty*, ed. by A.I. Miller (Plenum, New York), pp. 259–280.

Primas H. (1992): Time-asymmetric phenomena in biology. Complementary exophysical descriptions arising from deterministic quantum endophysics. *Open Systems & Information Dynamics* **1**, 3–34.

Primas H. (1994): Endo- and exo-theories of matter. In *Inside Versus Outside*, ed. by H. Atmanspacher and G.J. Dalenoort (Springer, Berlin), pp. 163–193.

Primas H. (1997a): The representation of facts in physical theories. In *Time, Temporality, Now*, ed. by H. Atmanspacher and E. Ruhnau (Springer, Berlin), pp. 241–263.

Primas H. (1997b): Individual description of dynamical state reductions in quantum mechanics. Unpublished report.

Selleri F., ed. (1988): *Quantum Mechanics Versus Local Realism* (Plenum, New York).

Sewell G.L. (1986): *Quantum Theory of Collective Phenomena* (Clarendon, Oxford).

Shaw R. (1981): Strange attractors, chaotic behavior, and information flow. *Z. Naturforsch.* **36a**, 80–112.

Sinai Ya.G. (1959): On the notion of entropy of a dynamical system. *Dokl. Akad. Nauk SSSR* **124**, 768–771.

Srinivas M.D. (1980): Collapse postulate for observables with continuous spectra. *Commun. Math. Phys.* **71**, 131–158.

Suchanecki Z. (1992): On lambda and internal time operators. *Physica A* **187**, 249–266.

Takesaki M. (1970): Disjointness of the KMS states of different temperatures. *Commun. Math. Phys.* **17**, 33–41.

Tjøstheim D. (1976): A commutation relation for wide sense stationary processes. *SIAM J. Appl. Math.* **30**, 115–122.