Can Classical Epistemic States Be Entangled?

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Abstract. Entanglement is a well-known and central concept in quantum theory, where it expresses a fundamental nonlocality (holism) of ontic quantum states, regarded as independent of epistemic means of gathering knowledge about them. An alternative, epistemic kind of entanglement is proposed for epistemic states (distributions) of dynamical systems represented in classical phase spaces. We conjecture that epistemic entanglement is to be expected if the states are based on improper phase space partitions. The construction of proper partitions crucially depends on the system dynamics.
Although improper partitions have a number of undesirable consequences for the characterization of dynamical systems, they offer the potential to understand some interesting features such as incompatible descriptions, which are typical for complex systems. Epistemic entanglement due to improper partitions may give rise to epistemic classical states analogous to quantum superposition states. In mental systems, interesting candidates for such states have been coined acategorial states, and among their key features are temporally nonlocal correlations. These correlations can be related to the situation of epistemic entanglement.

Keywords: non-commuting operations, phase space partitions, dynamical entropy, incompatibility, symbolic dynamics, epistemic entanglement, acategorial mental states, temporal nonlocality

1 Introduction

It has been an old idea by Niels Bohr that central conceptual features of quantum theory, such as complementarity, are also of pivotal significance far exceeding the domain of physics. Although Bohr was always convinced of the extraphysical relevance of complementarity, he never elaborated this idea in concrete detail, and for a long time after him no one else did so either.

By now, a number of research programs have been developed in order to pick up Bohr's proposal with particular respect to psychology and cognitive science. The first steps in this direction were made by the group of Aerts in the early 1990s (Aerts et al. 1993), using non-distributive propositional lattices to address quantum-like behavior in non-quantum systems. Alternative approaches have been initiated by Khrennikov (1999), focusing on non-classical probabilities, and Atmanspacher et al. (2002), outlining an algebraic framework with non-commuting operations. Two other, more recent lines of thinking are due to Primas (2007), addressing complementarity with partial Boolean algebras, and Filk and von Müller (2008), indicating strong links between basic conceptual categories in quantum physics and psychology.

Intuitively, it is quite unproblematic to understand why non-commuting operations or non-Boolean logic should be relevant, even inevitable, for mental systems that have nothing to do with quantum physics. Simply speaking, the non-commutativity of operations means nothing else than that the sequence, in which operations are applied, matters for the final result. And non-Boolean logic refers to propositions that may have unsharp truth values beyond yes or no, shades of plausibility or credibility as it were. Both versions obviously abound in psychology and cognitive science (and in everyday life), and they have led to well-defined and specific theoretical models with empirical confirmation and novel predictions. Five kinds of psychological phenomena have been addressed so far: (i) decision processes, (ii) semantic networks, (iii) bistable perception, (iv) learning, and (v) order effects in questionnaires (see Atmanspacher 2011, Sec. 4.7, for a compact review).

In earlier publications (beim Graben and Atmanspacher 2006, 2009) we studied in detail how the concept of complementarity can be sensibly addressed in classical dynamical systems as represented in a suitable phase space. The formal key to such a generalized version of complementarity lies in the construction of phase space partitions, which give rise to epistemic states. Descriptions based on partitions are compatible only under very specific conditions, otherwise they are incompatible or complementary. In this paper we ask whether entanglement, another central feature of quantum theory, may also be given meaning in the same framework.

2 Non-Commutative Operations

Non-commutative operations are at the core of quantum physics, where they appear as elements of algebras of observables. But non-commutative operations also abound in classical physical systems, as has been discussed frequently (see a recent paper by beim Graben and Atmanspacher (2006) including references given therein). A significant field in which this has become apparent is the theory of complex dynamical systems in physics.

Particularly fertile playgrounds for non-commutativity are complex systems outside physics for which interactions with their state (expressed as actions of an operator) are explicitly known to inevitably change that state. This is invariably the case in psychology: every interaction with a mental state changes that state in a way making it virtually impossible to prepare or re-prepare mental states strictly identically.

An intuitively appealing characterization of non-commutative operations A and B is to say that the sequence, written as multiplication, in which A and B are applied to a state makes a difference:

$$AB \neq BA\,. \tag{1}$$

If an addition of operations is defined as well, one can write:

$$[A, B] = AB - BA \neq 0\,, \tag{2}$$

and, given a commutator C, we have:

$$[A, B] = AB - BA = C \tag{3}$$

In quantum physics, the commutator for canonically conjugate quantum observables is universal: $C = h \cdot \mathbb{1}$, with h as the Planck action. For complex physical systems, and even more so for mental systems, we can hardly expect the commutator to be universal, but we may hope to find regularities for equivalence classes of systems. At present, we do not know how to do this in a deductive theoretical fashion, but there is a possibility to approach the problem empirically.

Commutation relations between two non-commuting operations A, B generically entail an uncertainty relation

$$\Delta A \cdot \Delta B \geq 1/2 |\langle C \rangle|\,, \tag{4}$$

where ΔA and ΔB are the variances of measured distributions of A and B, and $\langle C \rangle$ is the expectation value of C. Changing the conditions under which A and B are measured, it should be possible to investigate how the variances covary, and thus (at least) to estimate a lower bound for $\langle C \rangle$.

For the representation of commutation relations, i.e. of the way in which operators act on states, we need to specify a representation space. While this is typically chosen as a Hilbert space in quantum physics, a preferrable option for classical systems is a symplectic phase space or, more generally for complex systems, even a phase space without symplectic structure. In this contribution, we refer to the notion of a phase space in this general sense.

3 Phase Space Partitions

In the theory of dynamical systems, the state of a system is usually represented by a subset of its phase space Ω. For classical systems, their *ontic state* at a given time t is represented by a point $x \in \Omega$, while an *epistemic state* can be represented as a region $A \in \Omega$ comprising many ontic states.[1] More formally

[1] More precisely, epistemic states are distributions in a probability space over Ω, but for the present discussion it is sufficient to consider their support $A \in \Omega$; see beim Graben and Atmanspacher (2006, 2009). For a detailed discussion of ontic and epistemic states see Primas (1990) and Atmanspacher and Primas (2003) or, as a related framework, Spekkens (2007) and Harrigan and Spekkens (2010).

speaking, epistemic states are subsets $A_1, A_2, ..., A_n$ of Ω with $A_i \cap A_j = \emptyset$ and $\bigcup_{i=1,...,n} A_i = \Omega$.

If f is an observable of the system considered, then f ascribes a valuation to its states. For ontic states x, this valuation is simply $f(x)$, but for epistemic states A_i the situation is different: in the simplest case their valuation f is the same for all ontic states in the same subset A_i: $f(x) = f(y)$ for all ontic states $x, y \in A_i$. In this case, x and y are *epistemically equivalent* with respect to f.

The set $\mathcal{F} = \{A_1, A_2, ..., A_n\}$ of all subsets A_i is called a *phase space partition.*

- If every A_i is a singleton, i.e. represents an ontic state, $\mathcal{F}$ is the identity partition $\mathcal{I}$.
- If $A_1 = \Omega$, i.e. the entire phase space, $\mathcal{F}$ is the trivial partition.
- If $\mathcal{F}$ and $\mathcal{G}$ are finite partitions, $\mathcal{P} = \mathcal{F} \vee \mathcal{G} = \{A_i \cap B_j\}$ is a product partition.

4 Dynamics

Let us now consider the time evolution of the system, i.e. its dynamics, generated by a flow operator Φ acting on an ontic state x_t at time t,

$$x_{t+1} = \Phi^{t+1}(x_o) = \Phi(\Phi^t(x_o)) = \Phi(x_t)\,, \tag{5}$$

and combine this dynamics with the action of an observable f. The valuation $f(x_o)$ applies to an ontic state x_o in the epistemic state $A_{i_o} \in \mathcal{F}$. Similarly, $f(x_1) = f(\Phi(x_o))$ applies to an ontic state x_1 in the epistemic state $A_{i_1} \in \mathcal{F}$. This way, measuring $f(x_1)$ yields information about x_o, namely that x_o is contained in the epistemic state given by the intersection of A_{i_o} with the pre-image of A_{i_1}, $A_{i_o} \cap \Phi^{-1}(A_{i_1})$. We can continue this procedure iteratively up to measurements of x_n and obtain the information which measuring $f(x_n)$ yields about all previous states $x_{i<n}$.

Rather than talking about pre-images Φ^{-t} of epistemic states A_i, we generalize the terminology and refer to pre-images of the partition as a whole, $\Phi^{-1}(\mathcal{F}) = \{\Phi^{-1}(A_i)\}$. This allows us to define the dynamic refinement of $\mathcal{F}$ as a product partition $\mathcal{F} \vee \Phi^{-1}(\mathcal{F})$. The finest refinement $R\mathcal{F}$ is obtained in the limit $t \to \pm\infty$:

$$R\mathcal{F} = \bigvee_{t=-\infty}^{\infty} \Phi^{-t}(\mathcal{F}) \tag{6}$$

If $R\mathcal{F} = \mathcal{I}$, the partition $\mathcal{F}$ is the *generating partition* $\mathcal{P}_g$. It is distinguished by the fact that measurements of f yield complete information about the ultimate pre-image x_o of all epistemic states and, thus, gives rise to the determination of x_o as a dispersion-free ontic state. If $R\mathcal{F} \neq \mathcal{I}$, no dynamic refinement leads to such dispersion-free states.

5 Dynamical Entropy

For a partition $\mathcal{F} = (A_1, A_2, ..., A_n)$ of a state space Ω, a simple version of the *entropy* of the system is the well-known Shannon entropy

$$H(\mathcal{F}) = -\sum_{i=1}^{n} \mu(A_i) \ \log \mu(A_i)\,, \tag{7}$$

where $\mu(A_i)$ is the probability that the system state resides in partition cell A_i.

The *dynamical entropy* of a system in Ω requires us to consider its dynamics $\Phi : \Omega \to \Omega$ with respect to a partition $\mathcal{F}$:

$$H(\Phi, \mathcal{F}) = \lim_{n\to\infty} \frac{1}{n}\ H(\mathcal{F} \vee \Phi\mathcal{F} \vee ... \vee \Phi^{n-1}\mathcal{F}) \tag{8}$$

In other words, the dynamical entropy is the limit of the Shannon entropy of the product partition of increasing dynamical refinement.

An important upper bound for the dynamical entropy is the *Kolmogorov-Sinai entropy* (Kolmogorov 1958, Sinai 1959). It is defined as the supremum of the dynamical entropy over all partitions $\mathcal{F}$,

$$H_{KS} = \sup_{\mathcal{F}} H(\Phi, \mathcal{F})\,, \tag{9}$$

and it is assumed if $\mathcal{F}$ is a *generating partition* $\mathcal{P}_g$, so that $H_{KS} = H(\Phi, \mathcal{P}_g)$. If $\mathcal{F}$ is not generating, $H(\Phi, \mathcal{F}) < H_{KS}$.

Maximizing the dynamical entropy, $\mathcal{P}_g$ minimizes correlations among partition cells such that only correlations due to the dynamics Φ itself contribute to $H(\Phi, \mathcal{P}_g)$. This can be understood due to the fact that points on boundaries between cells (epistemic states) A_i are (roughly) mapped onto points on boundaries between cells A_i. As a consequence, $\mathcal{P}_g$ is dynamically stable, the definition of the corresponding epistemic states is robust under the dynamics, and spurious correlations due to blurring cells are excluded.

The concept of a generating partition is related to the concept of a *Markov chain* in the theory of stochastic systems. Every deterministic system of first order gives rise to a Markov chain which is generally neither ergodic nor irreducible. Such Markov chains can be obtained by so-called *Markov partitions* that exist for expanding or hyperbolic dynamical systems (Sinai 1968, Bowen 1970, Ruelle 1989). For non-hyperbolic systems no corresponding existence theorem is available, and the construction can be even more tedious than for hyperbolic systems (Viana *et al.* 2003). For instance, both Markov and generating partitions for nonlinear systems are generally non-homogeneous, i.e. their cells are typically of different size and form.[2]

[2] Every Markov partition is generating, but the converse is not necessarily true (Crutchfield 1983, Crutchfield and Packard 1983). For the construction of generating partitions from empirical data it is often more convenient to approximate them by Markov partitions (Froyland 2001, Allefeld et al. 2009).

6 Symbolic Dynamics

Since generating partitions are stable under the phase space dynamics Φ, they can be used to construct symbol sequences s in a symbolic representation space S in such a way that s is *topologically equivalent* to Φ.[3] This idea is exploited in the field of symbolic dynamics (Lind and Marcus 1995), where a continuous mapping $\pi : \Omega \to S$, called an *intertwiner*, is defined whose inverse π^{-1} exists and is also continuous. Then, the dynamics of epistemic states in Ω can be faithfully expressed as a symbol sequence $s \in S$ by:

$$\Phi = \pi \circ s \circ \pi^{-1} \tag{10}$$

If the epistemic states A_i in Ω are cells of a generating partition, the intertwiner π exists, and s and Φ are guaranteed to be topologically equivalent. This means essentially that “neighboring” epistemic states in Ω will be mapped onto “neighboring” states in S. The construction of $\mathcal{P}_g$ entails that differences between epistemically equivalent ontic states in Ω are deliberately disregarded.

Partitions that are not generating lead to symbolic dynamics deviating from perfect topological equivalence. Skufca and Bollt (2008) investigated how the corresponding deviation of the map from Ω to S from an intertwiner can be characterized quantitatively by a “homeomorphic defect”. This paves the way to specify the degree to which a symbolic description is a faithful representation of an underlying phase space dynamics.

Note that the concept of topological equivalence differs from *topological conjugacy* if the dynamics is continuous in time. Topological conjugacy requires an intertwiner mapping individual trajectories, i.e. ontic states defined pointwise in Ω, which can be parametrized pointwise in time. By contrast, epistemic states $A_i \in \Omega$ have no individual trajectories but sets of trajectories, so that π cannot map phase space states A_i onto symbolic states s together with a one-to-one mapping of their time parameter. This motivates topological equivalence as a relation weaker than topological conjugacy.

7 Improper Partitions

For improper partitions that are not generating, Bollt et al. (2001) coined the notion of “misplaced” partitions. Their use to determine the Kolmogorov-Sinai entropy leads to a systematic underestimation, because the cells of misplaced partitions are not stable under the dynamics and, thus, entail blurring effects of cell boundaries effectively violating the disjointness of epistemic states. As a

[3] While the construction of symbolic descriptions based on generating partitions is essentially motivated by their *stability* under the dynamics, a viable alternative relies on *information* theoretical ideas. This alternative is embedded in the framework of computational mechanics, as pioneered by Crutchfield and coworkers. See Crutchfield and Shalizi (2001) for a comprehensive review, and Shalizi and Moore (2003) for relations between their and our approach.

consequence, there will be "spurious" correlations in addition to those originating from the dynamics itself. These "spurious" correlations obviously arise from epistemic states, not from decompositions of ontic entangled states. Therefore they differ drastically from entanglement correlations as exhibited by entangled quantum systems (cf. Atmanspacher and Primas 2003).

Although misplaced partitions are undesirable for extracting the Kolmogorov-Sinai entropy of a system or for defining faithful symbolic representations of the system dynamics by topologically equivalent symbol strings, they may be interesting for other purposes. For instance, they imply non-Boolean features arising from coarse grainings of purely classical phase spaces (cf. Westmoreland and Schumacher 1993). In other words, improper partitions may lead to a multitude of symbolic descriptions that are (all or partly) incompatible with each other, yet being (all or partly) necessary for a complete picture of the system considered.[4]

This may be a reason (surely not the only one) why sciences dealing with situations far more complex than in physics show a profound tendency toward non-universal theoretical frameworks of thinking. If phase space partitions of complex systems are set up ad hoc, the likelihood to find a proper (generating) partition is extremely low, and incompatible descriptions are an almost certain consequence. Atmanspacher and beim Graben (2007) argued along those lines for symbolic psychological descriptions and proposed a way to construct such descriptions based on proper partitions of neural phase spaces. A pertinent example of such a construction was demonstrated by Allefeld et al. (2009).

8 Compatibility and Other Relations between Partitions

For a brief summary of possible relations between partitions we consider two observables f and g inducing epistemic states according to partitions $\mathcal{F}$ and $\mathcal{G}$. Then we can define the following relations (cf. beim Graben and Atmanspacher 2009).

- Two partitions $\mathcal{F}$ and $\mathcal{G}$ are *compatible* if and only if they are both generating, $R\mathcal{F} = R\mathcal{G} = \mathcal{I}$. This means that every ontic state x_o is epistemically accessible as a pre-image $\Phi^{-t}(\mathcal{F}, \mathcal{G})$.
- Two partitions $\mathcal{F}$ and $\mathcal{G}$ are *incompatible* if at least one of them is not generating, $R\mathcal{F} \neq R\mathcal{G}$.
- Two partitions $\mathcal{F}$ and $\mathcal{G}$ are *complementary, or maximally incompatible*, if their finest refinements are disjoint, $R\mathcal{F} \cap R\mathcal{G} = \emptyset$.
- Two partitions $\mathcal{F}$ and $\mathcal{G}$ are *comparable* if $R\mathcal{F}$ is a refinement of $R\mathcal{G}$ or vice versa. This entails that compatibility implies comparability. Even incompatible partitions may be comparable, if one of them is generating.
- Two partitions $\mathcal{F}$ and $\mathcal{G}$ are *commensurable* if a common language $T(\mathcal{U})$ embedding $T(\mathcal{F})$ and $T(\mathcal{G})$ exists (cf. Primas 1977) such that $R\mathcal{U}$ is a refinement of $R\mathcal{F}$ and $R\mathcal{G}$. Comparability implies commensurability.

[4] Primas (2007) proposed the formal framework of *partial Boolean algebras* to refer to locally Boolean propositional lattices pasted together in a non-Boolean fashion.

9 Epistemic Entanglement

An interesting implication of improper, misplaced partitions is that they produce coarse grainings that change dynamically, thus yielding correlations in the dynamics of the system that are not a result of the dynamics itself but of overlapping coarse grains. For reasons mentioned above, such correlations are undesirable in symbolic dynamics and ergodic theory. However, they produce features that may look phenomenologically like entanglement correlations insofar as they are not explainable in terms of causal interactions of a system (e.g., with its environment).

This could provide insight concerning particular quantum-like features in classical systems, e.g. "Brownian entanglement" as reported by Allahverdian et al. (2005). Two particles undergoing Brownian motion were shown to create correlations analogous to quantum entanglement for coarse-grained velocities. From the perspective of our approach, it may be conjectured that this coarse-graining yields improper partitions inducing the correlations in question. Allahverdian et al.'s observation that the correlations disappear for an increasingly refined resolution of the coarse-graining points to an asymptotic epistemic accessibility of classical ontic particle states in their study.

Since ontic entanglement, as in genuinely entangled quantum systems, does not depend on measurement resolution or other partitioning issues, varying correlations due to altered partitions are a clear indicator for *epistemic entanglement*. This raises the question of whether it might be possible to adjust the degree of such epistemic entanglement in a controlled way. To our knowledge, this has not been studied so far, and at present we can only speculate about this possibility and its potential value. In the remaining sections we will sketch some corresponding ideas.

10 Acategorial Mental States

A state exhibiting epistemic entanglement according to blurred boundaries as discussed above would be a state represented by the intersection $A_i \cap A_j$ of non-disjoint states A_i and A_j. In a way, such a "superposition" state shares features of both A_i and A_j. On the other hand, neither A_i nor A_j is actualized because the actual state resides somehow "in between" them, offering the potential to actualize either one or the other state. Needless to say, this resembles the idea of a "reduction" of a quantum superposition state very closely.

An application of this idea to mental states was proposed by Atmanspacher (1992) and recently elaborated by Atmanspacher and Fach (2005) and Feil and Atmanspacher (2010). The present mainstream understanding of mental activity is framed by mental representations (or categories), which have been learned and stored, and which can be actualized by suitable stimuli (cf. Metzinger 2003). Mental states that actualize such representational categories are temporarily stable categorial states.

The notion of *acategorial states*, taken from Gebser (1986), has been used to address intermediate phases, for instance phases during which the mental state transits from one categorial state to another. The possibility of acategorial states depends crucially on the presence of established representations, none of which is actualized by an acategorial state though. While categorial states reside in stable mental representations strictly distinguishable from one another, inherently unstable acategorial states reside between adjacent categorial states and hold the possibility to relax into each one of them.[5]

Categorial states can be represented as epistemic states in appropriate phase spaces (Atmanspacher 1992, Feil and Atmanspacher 2010), and it is a challenging speculation to conceive of acategorial states as states exhibiting epistemic entanglement as indicated above. How might the experience of such states be like? A pertinent remark by Sudarshan (1983), responding to the question of how quantum states might be "perceived directly", proposes a mode of awareness in which

> "sensations, feelings, and insights are not neatly categorized into chains of thoughts, nor is there a step-by-step development of a logical-legal argument-to-conclusion. Instead, patterns appear, interweave, coexist; and sequencing is made inoperative. Conclusions, premises, feelings, and insights coexist in a manner defying temporal order."

11 Temporal Nonlocality

From a slightly different perspective, recent work by Atmanspacher and Filk (2010) on bistable perception suggests that the phenomenology described by Sudarshan (1983) may be related to the violation of temporal Bell inequalities entailing temporally nonlocal correlations.[6] It is a necessary condition for such a violation that the dynamics of the system considered is governed by operators that do not commute.

The resulting *temporal nonlocality* of mental states can be interpreted such that these states cannot be sharply (pointwise) localized along the time axis, and their characterization by sharp (classical) observable variables is inappropriate. Rather, temporally nonlocal states appear to be "stretched" over an extended time interval whose length may depend on the specific system considered. Within this interval, relations such as "earlier" or "later" are illegitimate designators of the system state. This is just another way of saying that it is impossible to define causal relationships within such a time interval (Filk and von Müller 2009).

It is tempting to relate this *temporal nonlocality* to a "window of temporal nowness", a concept that transcends a sharp boundary of presence between past

[5] By contrast, non-categorial states would be states without established representations. Feil and Atmanspacher (2010) suggested that acateogrial and non-categorial states are two different variants of the currently much discussed philosophical notion of "non-conceptual mental content" (Bermúdez and Cahen 2008).

[6] See also Atmanspacher and Filk (2011). While the original Bell inequalitites and their associated effects of nonlocality are usually discussed in terms of spatial relations between spatial subsystems, temporal Bell inequalities refer to relations between temporal segments of the history of a system.

and future (Filk and von Müller 2009, Pöppel 1997). However, the idea itself is much older and dates back at least to James' notion of the "specious present", a present mental state extending over a time interval rather than fixed to an instant of vanishing duration.

Acategorial states are interesting candidates for temporal nonlocality as a property of mental states. Their intrinsic instability can easily be related to an indeterminate location in time that effectively amounts to their temporal extension. Presently we do not know whether and how it might be possible to actively control the temporal extent of such states. Considering them as epistemically entangled states according to Section 9 could provide theoretical access to this question.

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