

Deviations from Physical Randomness Due to Human Agent Intention?

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Abstract

Advanced techniques for pattern detection and time series reconstruction are applied to look for deviations from randomness in random time series generated under different conditions. Based on a detailed error analysis, evidence for significant non-random contributions is found under conditions in which human agents are asked to carry out intentional tasks effectively corresponding to deviations from randomness in the output of a random physical device. There are strong indications that the observed non-random deviations are related to enhanced temporal correlations.

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1 Introduction

1.1 General

The question of relationships between the material world and its apparently non-material counterpart or complement, respectively, belongs to the oldest, most puzzling, and most controversial questions in both the history of philosophy and the history of science. One of the main reasons for its controversial nature is that the question itself is understood in different ways depending on basic assumptions concerning our conception of reality. What makes all approaches toward this question as well as the discussions about those approaches so difficult is the fact that those assumptions are most often implicit rather than explicitly clarified.

From the viewpoint of a philosophically informed contemporary physicist, there are two general conceptual frameworks within which reality can be conceived. (For more details about these topics the reader is referred to the relevant literature, e.g., Chalmers (1996).) One of them is typically denoted as *physicalism* (or *materialism*) and expresses the idea that the basis of reality consists of the material world alone; anything like qualia, consciousness, psyche, mind, or spirit is based on the material elements and fundamental laws of physics. For physicalists, the way in which these apparently non-material higher-level properties can be explained is a follow-up question, again answered differently within different ways of thinking, using conceptual schemes such as, e.g., emergence, supervenience, or reduction. These concepts are tightly related to each other, and many of those authors who take the care to define them at all define them differently.

The other general framework is characterized as *dualism*, ranging from ontological to epistemological and methodological versions. Briefly speaking, ontological dualism maintains that the world *consists* of mind and matter (or other, corresponding concepts) as ultimately separate “substances”. Epistemological dualism refers to mind and matter as fundamentally different domains with respect to our modes of gathering and processing knowledge of the world, irrespective of what this world “as such” (“in itself”) may or may not be. Methodological dualism reflects an attitude that is neutral to the claims made by the other two variants. It utilizes the mind-matter distinction as a basic, but maybe not the only applicable methodological tool to inquire into the structure of the world. In its weakest (methodological) form, dualism is a prerequisite of any physicalist approach insofar as the latter presupposes a distinction between matter and something that appears to be non-material and – in one way or another – has to be related to, explained by, or even derived from the elements and laws of the material world.

How can such a relationship, explanation, or derivation be understood in detail? Again, quite a number of different conceptions exist in this regard. They are related to terms such as correlation, interaction, causation, influence and the like. Unfortunately, the popularity of those terms in all kinds of publications is much higher than the precision with which they are defined and applied. For this reason,

many such publications are effectively worthless for any sound understanding. It is a mandatory condition for substantial and responsible interdisciplinary work that concepts and notions be carefully defined when they are used. If this important first step in scientific work is disregarded, we can neither decently communicate nor can we hope to contribute to any reasonable progress whatsoever.

In this paper, we presuppose the minimal version of methodological dualism as indicated above. Beyond this, any option for stronger dualisms or one of the various kinds of physicalism remains open. The investigations that will be presented in the following address the question whether the behavior of a physically random system can be “intentionally perturbed” by human agents. The most general sense in which the notion of a “perturbation” is to be understood here refers to an experimental design that will be described in the following subsection. Roughly speaking, the question is whether the randomness of a physically random system changes when human agents are asked to carry out certain “intentional tasks” that are defined so as to be related to deviations from randomness in the behavior of the system. We address this question without assuming psychophysical correlations or forwarding speculations with respect to relationships between matter and mind. We do not presuppose any concepts of correlations, interactions, and causal or other influences between material systems and non-material (mental) systems.

Randomness is itself a concept that needs to be considered in more detail. First, there is the much discussed question whether randomness or chance is an “ontic” property of the material world, without any reference to the knowledge of observers, or whether it is an “epistemic” expression of our limited knowledge about an ontic world. Second, there are different types of probability distributions for random events generated by different types of stochastic processes (e.g., binomial, exponential, Poisson, Gauss, equi-distributions, etc.). In the following we use the notion of a random distribution as an abbreviation for a probability distribution of random events.

Another most obvious problem with which any working scientist is familiar arises from the fact that the mathematical, measure theoretic definition of probability a la Kolmogorov implies the limit of infinitely many identical and independent realizations of an event. In practice, this is impossible to achieve. We always deal with finitely many events, they are never identical in every respect, and it has to be checked carefully whether and in what sense they are really independent from each other.

The finiteness of any empirical collective of events has the consequence that many mathematical theorems about probabilities (based on the limit $N \rightarrow \infty$) must be applied with caution. For instance, in the area of complex systems research many examples are known for which limit theorems (e.g., laws of large numbers) are not naively applicable and ergodicity must not be implicitly presupposed. Novel approaches and techniques of modern statistics such as second order statistics or large deviations statistics (cf. Atmanspacher et al. 1997) offer new insights beyond a standard characterization of a distribution in terms of its first two moments. For

appropriate methods of analyzing finite collectives in such situations, it has proven to be necessary to characterize the concept of randomness with reference to the range of N with which one is empirically dealing.

Another important topic is the independence of events as members of a random collective. It is well known that distributions of states of a system can be perfectly random, e.g., in the sense that they are perfectly represented by a Gaussian distribution, but nevertheless there can be non-random features, e.g., correlations, as far as the transition probabilities between states (i.e., the dynamics of the system) are concerned. In such a case, individual events are not independent. They depend on their prehistory and show significant temporal correlations. Correlations are significant if they deviate (at a level to be specified) from the size of fluctuations that are expected due to the finite number of events. If the transition probabilities are random in the sense of an equidistribution, then the state probabilities are equidistributed as well, and the behavior of the system is called doubly stochastic. A number of measures that characterize and distinguish these different kinds of randomness have been analysed and compared with each other by Wackerbauer et al. (1994). For other recent studies specifically devoted to dynamical randomness see Gaspard and Wang (1993) and Deco et al. (1998).

In the present article, we apply a procedure known as scaling index analysis to the time series obtained from a physical random event generator (REG) under two different conditions: 1. The output of the REG is sampled without any additional constraints on the experimental setup; the corresponding data are used for calibration; 2. The output of the REG is analysed for situations in which human agents are asked to carry out “intentional tasks” that are defined so as to be related to non-random contributions to the activity of the REG. The applied analysis is capable of detecting very faint deviations from a random distribution of transition probabilities even if the distribution of state probabilities does not significantly deviate from randomness.

In the following subsection 1.2 the experimental setup and protocol are briefly described. Section 2 gives an introduction to scaling exponents, scaling indices, and their application to time series. Section 3 describes in detail how errors for the results of such an analysis can be estimated. In Section 4, a scaling index analysis is applied to a small set of data involving human agent intention. Section 5 summarizes the main results.

1.2 Experimental

The prototype of the experimental setup from which the analyzed data are obtained goes back to Schmidt (1970). It has been refined and applied to a large range of empirical questions in the work of the Princeton Engineering Anomalies Research (PEAR) project within the last two decades. The general question behind that work is whether some “intentional activity” of human agents changes the output of physical systems which are expected to produce random events. The work of

PEAR suggests that there are significant deviations from randomness when human agent intention is involved (Jahn et al. 1997, see also Utts 1991). A replication of the PEAR studies has been started at the Institut für Grenzgebiete der Psychologie und Psychohygiene (IGPP) at Freiburg in 1996. The present investigations refer to a small subset of data obtained in the IGPP replication study.

The material core of the experiment is a random event generator (REG) whose output (after some processing) consists of binary sequences. In the IGPP replication study, a portable random event generator has been utilized that is different from the REG originally developed and used by PEAR. More details concerning these two sources of randomness can be found in Nelson et al. (1989) and Bradish (1993). Both sources are semiconducting devices, producing a mixture of quantum and thermal noise. After two hundred bits (0s and 1s) have been generated by the REG, the number of 1s is counted and the result is taken as the outcome of a single “trial” for the experiment.

Sequences of 100 and 1000 trials, respectively, constitute experimental “runs” consisting of successive integer “raw data” x_i scattered around an expected mean value of 100 with an expected standard error of $\sqrt{50}$. The trials of a run can be graphically represented (depending on feedback options) on a monitor. This representation ξ_i is obtained from the raw data by

$$\xi_i = \sum_{j=1}^i x_j - 100i \quad (1)$$

or, recursively, by

$$\begin{aligned} \xi_1 &= x_1 - 100, \\ \xi_i &= \xi_{i-1} + (x_i - 100) \quad \text{for } i > 1. \end{aligned} \quad (2)$$

The series ξ_i is thus cumulative in the sense that a constant value above (below) 100 in the raw data would produce a monotonically increasing (decreasing) graph for ξ_i on the monitor. If the REG produces random numbers, the overall expectation (for $N \rightarrow \infty$) is that the curve ξ_i does not significantly deviate from the baseline 0.

According to the experimental protocol, human agents are asked to intentionally try to achieve that the curve rises above or falls below the baseline, or to be intentionally neutral with respect to the appearance of the curve in different runs. (They are *not* explicitly asked to achieve deviations from randomness.) The corresponding “modes of intention” are denoted as “high”, “low”, and “baseline”. It is left to the agents how to realize each of these modes of intention cognitively. The selection of a particular intention per run can be decided randomly or by the agents themselves. An experimental “session” consists of 1000 trials per each intention. For runs consisting of 100 (1000) trials, a session therefore amounts to 10 runs (1 run) per intention, i.e., 30 (3) runs in total. For the analysis described in Sec. 4, data from agents with 10 sessions each have been used, i.e., 10000 trials per agent and intention. To avoid artificial correlation effects, the analysis has been based on the raw data x_i rather than the cumulative data ξ_i .

The most prominent result obtained from analyses of the data pool collected by PEAR is that the mean value of the distribution of state probabilities (integer values x_i) shows small systematic shifts above and below the expected mean of 100.0 for experimental conditions with high and low intention, respectively. The precise values as given by Tab. 1 in Jahn et al. (1997) are 100.026 for high, 99.984 for low, and 100.013 for baseline. Although the corresponding high-low separation of 0.042 is tiny, the overall z -score for the entire set of individual sessions is 3.81, with a p -value of 7×10^{-5} . (For more details on the importance of effect sizes in this and related studies see Utts (1991) and Delanoy (1996)).

The analysis presented in the following differs from the analysis by PEAR insofar as it allows us explicitly to address transition probabilities between states together with state probabilities. As mentioned above, the data used for this purpose are data from the IGPP replication study. This study is not yet finished, implying that any assignment of individual runs to agent intentions is kept hidden so far. Therefore, our analysis is strictly bound to an analysis of deviations from randomness and does not refer to specific agent intentions.

2 Scaling Index Analysis

2.1 Scaling Exponents and Scaling Indices

Any statistical characterization of a system in terms of its subsystems, e.g., idealized by N points in some suitable state space, applies more or less directly the concept of correlations (or covariances). Since correlations formally represent correction terms to a constant density, vanishing correlations characterize a translationally invariant distribution, i.e., a distribution that is homogeneous with respect to its density.

Many systems with natural distributions (in state space or in the usual three-dimensional position space) show a scale-free, “self-similar” behavior of correlations. In this case, the correlation decrease as a function of increasing distance can be quantified by a constant exponent. If the scaling exponents are identical for each reference point within the system, one has a specific kind of “conditional homogeneity” (Mandelbrot 1982) with respect to the scaling properties of the system. If the scaling exponents are identical for each direction in the relevant space, then the distribution is rotationally invariant with respect to its density for each reference point, i.e., locally isotropic. If the scaling exponent is identical for correlations of arbitrary order, the considered system is a so-called “single fractal” in the limit $N \rightarrow \infty$. For different exponents with different orders of correlations the system is a multifractal for $N \rightarrow \infty$. Insofar as correlation functions can be much more involved than simply showing exponential decay, fractals or multifractals are systems with relatively unsophisticated statistical properties.

For numerous applications as well as for the basic formalism of multifractals the reader is referred to a review by Paladin and Vulpiani (1987). Basically, there are two main lines of applications: (i) the statistical characterization of state space

representations of dynamical systems; (ii) the statistical characterization of complex objects in some suitably chosen space, including position space. Techniques that have been developed in the context of (ii) frequently refer to the detection of patterns in front of some random background. For examples, see Atmanspacher et al. (1989), Atmanspacher et al. (1995), Morfill et al. (1994), R ath and Morfill (1997), Wiedenmann et al. (1997). Concerning (i), the most prominent class of applications is the characterization of attractors of dissipative systems in terms of static and dynamic invariants. In our present context, a combination of both kinds of application will be used. The procedure has been registered as patent # EP 0 700 544 B1 at the European Patent Office¹ and will be described subsequently.

If a point set \mathcal{S} is a (multi-) fractal system on which a natural measure μ is defined, one can choose a partition into subsets \mathcal{S}_α such that

$$\mathcal{S} = \bigcup_{\alpha} \mathcal{S}_{\alpha}. \quad (3)$$

The quantity α is a continuous variable characterizing the local scaling properties of the set. Its meaning is that of a local scaling exponent

$$\alpha_i = \lim_{\epsilon \rightarrow 0} \frac{\log p_i(\epsilon)}{\log \epsilon} \quad (4)$$

within the i th of $M(\epsilon)$ boxes (spheres) with edge length (diameter) ϵ required to cover \mathcal{S} . (Using boxes (spheres) corresponds to using the maximum (Euclidean) norm to determine distances between points.) As usual, the probability $p_i(\epsilon)$ is defined by the measure on \mathcal{S} :

$$p_i(\epsilon) = \int_{\text{box } i} d\mu(\mathbf{x}). \quad (5)$$

The scaling exponent measures how fast the number of points within a box decreases as ϵ is reduced. It therefore measures the ‘‘strength of a singularity’’ (for $\epsilon \rightarrow 0$). If $\alpha_i = \alpha$ for all i , then the point set is a single fractal. The set of all scaling exponents α_i can be compactly represented by a so-called ‘‘spectrum of singularities’’ $f(\alpha)$ (Halsey et al. 1986).

For all practical purposes, the limit $\epsilon \rightarrow 0$ (as well as some thermodynamic limit of an infinite number of points $N \rightarrow \infty$) is inaccessible. Assuming local isotropy within each box, one can introduce a ‘‘crowding index’’ (Grassberger et al. (1988)) or ‘‘scaling index’’

$$\alpha_i = \frac{\log N_i(\epsilon_2) - \log N_i(\epsilon_1)}{\log \epsilon_2 - \log \epsilon_1}, \quad (6)$$

¹The international patent number is PCT/EP94/01752, the U.S. patent application number is 08/549,711. Requests for release or use of software or other detailed information have to be directed to and negotiated with Garching Innovation GmbH (K oniginstr. 19, D-80539 M unchen). Garching Innovation administers patent rights held by the Max-Planck-Gesellschaft zur F orderung der Wissenschaften. For further advice beyond and after clarification of legal issues, in particular concerning practical expertise in applying the method to specific problems, please contact one of the authors (HA, HS).

where $N_i(\epsilon)$ is the number of points within a box i of size ϵ , to be considered around point i of the entire point set ($i = 1, 2, \dots, N_{\text{tot}}$), and $\epsilon_1 < \epsilon_2$. The scaling index is thus defined for a specific range of ϵ , which in turn defines a locality criterion with respect to which correlations are characterized by α_i . Counting those boxes (to be constructed around points) that give rise to α_i for all i , a histogram $N(\alpha)$ is obtained which is used instead of $f(\alpha)$ in the case of finitely many points. For more details, see Atmanspacher et al. (1989).

For perfectly random distributions in a d -dimensional space, the (ideal) $f(\alpha)$ spectrum is a δ function at $\alpha = d$. Due to the restrictions imposed by a finite number of points, non-zero ϵ , and finite binning of α , $N(\alpha)$ is broader and its mean is shifted toward values of α smaller than d . The same happens for regular patterns with topological dimension $n < d$, where $f(\alpha)$ is a δ function at $\alpha = n$. It is intuitively clear that a scaling index analysis can be useful to discriminate between non-random features with $n < d$ and a random background. For faint non-random contributions dominated by a random distribution such a task is difficult since – depending on their nature – they typically appear as small deviations in the left wing of an $N(\alpha)$ histogram characterizing randomness. For such situations, it is crucial to take care in selecting a good locality criterion (range of ϵ over which α_i is calculated) and in estimating errors in an appropriate manner.

2.2 Representation of Time Series with Delay Coordinates

If a correlation analysis based on a scaling index analysis is to be carried out for a time series (temporal sequence of data) rather than a spatial pattern, then it is necessary to represent the time series in a space with dimension greater than 1. A standard technique to achieve such a representation has been proposed by Packard et al. (1980) in order to reconstruct attractors of dissipative dynamical systems and extract their invariants. Our goal in the present study is less ambitious since we do not look for invariants of a real physical process but simply for correlations in the temporal sequence of data points in a time series. Hence, we do not have to check for stationarity or ergodicity – conditions that are necessary for a sound derivation of physically relevant invariants.

Inspired by Packard et al.’s delay coordinate technique, we use the original time series $\phi(t)$ to construct a number $(d - 1)$ of additional time series, each delayed by a temporal interval Δt with respect to its predecessor. In this way, new coordinates x_i are obtained according to

$$\begin{aligned} x_1(t) &= \phi(t) \\ x_2(t) &= \phi(t + \Delta t) \\ &\vdots \\ x_d(t) &= \phi(t + (d - 1)\Delta t), \end{aligned} \tag{7}$$

with

$$\vec{x}(t) = (x_1(t), x_2(t), \dots, x_d(t)). \tag{8}$$

In this manner, the original (one-dimensional) time series can be represented in a d -dimensional space, and the corresponding histogram $N(\alpha)$ can be calculated. This allows us to discriminate regular (deterministic) contributions from random noise in the behavior of dynamical systems. In particular, non-random features in the transition probabilities between individual members of the time series can be detected in addition to non-random state probabilities. Such a task is fairly straightforward in case of low-dimensional attractors (fixed points, limit cycles); compare, e.g., Siffling (1996).

For investigations of less prominent non-random temporal features in a random background process, the analysis becomes more sophisticated. In this case, one would ideally proceed to embedding dimensions d as high as possible since the α -range of $N(\alpha)$ characterizing random contributions increases with d whereas the α -range of the non-random part of $N(\alpha)$ drops back with increasing d . This means that faint non-random contributions appearing as deviations in the left wing of the dominating random part of $N(\alpha)$ become more pronounced and thus can be better discriminated as d is increased. However, severe restrictions on the size of d are imposed by the length of the original time series. For as few as 1000 data points, the point distribution resulting from the delay technique in $d = 4$ is certainly not dense enough to admit a statistically reasonable analysis. For 10000 data points, $d = 4$ is a reasonable upper limit for d .

3 Error Analysis

In experiments such as briefly sketched in Sec. 1.2, one should by no means expect easily detectable, major deviations from a perfectly random distribution using its $N(\alpha)$ histogram. For this reason, extremely careful error estimates are mandatory for a sound scaling index analysis. These error estimates have to be based on the same parameters as used in the analysis of those data taken under the influence of human agents. For time series consisting of 10000 data points $\Delta t = 1$ and $d = 4$ have been used. Scaling indices α are binned in steps of 0.01, where results are robust with respect to further changes in bin width.

For an optimal locality criterion, $\epsilon_1 = 4.6$ and $\epsilon_2 = 12.7$ have been determined by minimizing the differences between $N(\alpha)$ histograms for calibration data and $N(\alpha)$ histograms for a Gaussian distribution fitted to the full width of half maximum (FWHM) of the state distribution of calibration data as a function of possible ϵ -ranges. (Variations of ϵ produce changes that leave the significance level of deviations from randomness in individual data sets invariant within errors.) This means that we are looking for correlations on the mentioned distance scale only. Correlations on larger scales, e.g., approaching the diameter of the point distribution as a whole are left out of consideration. Calibration data are taken from the experimental random number device as described in Sec. 1.2.

In the following, we first describe how errors for a scaling index analysis can

be estimated from the experimental calibration data and from data satisfying a Gaussian state distribution fitted to the FWHM of the state distribution of the calibration data. Next, we compare the error estimates thus obtained with error estimates based on Monte Carlo type studies of random sequential permutations of the calibration data and of fitted Gaussian distributions. Note that all Monte Carlo studies use the same state probabilities as given by the (Gaussian or calibration) distribution to which the permutations are applied. Only transition probabilities between states are modified in the Monte Carlo simulations.

The relative differences (normalized with respect to the total number N_{tot} of points)

$$\delta_{1,2} = \frac{1}{N_{\text{tot}}} (N(\alpha)_1 - N(\alpha)_2) \quad (9)$$

between two histograms $N(\alpha)_1$ and $N(\alpha)_2$ will be shown and discussed in integral form according to

$$\Delta_{1,2} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (N(\alpha)_1 - N(\alpha)_2) d\alpha. \quad (10)$$

Such an integral representation² has the advantage that consistent trends in the differences over extended ranges of α become clearly visible even if the differences themselves are small. Values α_{ext} at which Δ is maximal (minimal) correspond to the onset of negative (positive) differences after an extended α -range of positive (negative) differences. Hence, an extremum in Δ indicates that those differences giving rise to it are at $\alpha < \alpha_{\text{ext}}$.

As an example, the dotted line in Fig. 1 shows the differences $\delta_{1,2}$ of two histograms. They are calculated for each α bin separately and normalized with respect to the total number N_{tot} of points. The solid line in Fig. 1 shows the corresponding integral plot of $\Delta_{1,2}$, again as relative deviations. For $\alpha < \alpha_{\text{ext}} = 3.18$, the differences $\delta_{1,2}$ are consistently negative. This trend changes at $\alpha_{\text{ext}} = 3.18$, where $\Delta_{1,2}$ is minimal and the differences $\delta_{1,2}$ become positive. The relative deviation at $\alpha_{\text{ext}} = 3.18$ is negative and amounts to $\Delta_{1,2} \approx 3.56\%$. Further aspects concerning the interpretation of Fig. 1 will be discussed in Sec. 4.

3.1 Calibration distributions and Gaussian distributions

The experimental device described in Sec. 1.2 produces sequences of integer random numbers whose mean is 100, scattering between, roughly speaking, 70 and 130. Considering these numbers as states, the state distribution of a hundred sets of calibration data, each consisting of 10000 data points, was determined as a histogram. (Note that this is *not* yet the histogram $N(\alpha)$ of scaling indices.) From the FWHM of this distribution, it is possible to fit a Gaussian distribution of states. Figure 2 shows the relative differences between the mean state distribution of calibration

²As always, integrals are numerically evaluated by a sum of finitely many terms, here given by the values of $N(\alpha)$ within bins of $\Delta\alpha = 0.01$.

data and the mean of 100 different realizations of Gaussian distributions with the same FWHM. Deviations are far below $\sqrt{N_{\text{tot}}}/N_{\text{tot}} = 1\%$.

Based on scaling index histograms $N(\alpha)$ for all 100 Gaussian distributions generated as just mentioned, one can now determine the relative integral differences between each individual Gaussian realization and the mean of all 100 realizations

$$\Delta_{\text{Gauss}} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (N(\alpha)_{\text{Gauss},i} - \langle N(\alpha)_{\text{Gauss}} \rangle) d\alpha. \quad (11)$$

The extrema of the integrated differences Δ_{Gauss} over α are crucial for the detection of deviations from randomness. Figure 3 shows the extrema of Δ_{Gauss} for all 100 realizations in the range $2.0 < \alpha < 5.0$. The distribution of extrema is fairly symmetrical with respect to positive and negative values. The mean value of these extrema over all scaling indices and all realizations ($i = 1, \dots, 100$) is given by

$$\langle \max |\Delta_{\text{Gauss}}| \rangle_{\alpha} = (1.15 \pm 0.72)\%. \quad (12)$$

As repeatedly indicated above, it will be essential for the detection of faint non-random contributions to have a sound significance criterion for deviations from a random $N(\alpha)$, particularly at its left wing. Recall that deviations from randomness contribute to those α that are smaller than some “mean” α at which $N(\alpha)$ has its maximum. Therefore, it is important to have an α sensitive error estimate, admitting a detailed error analysis with respect to selected ranges of α . Since the size of “random” fluctuations in a distribution with a finite number of points ($N_{\text{tot}} = 10000$) is a non-trivial function of α , suitable α sensitive error estimates have to be calculated on intervals $[0, \alpha_{\text{max}}]$. The resulting mean values

$$\langle \Delta(\alpha_{\text{max}}) \rangle = \langle \max |\Delta_{\text{Gauss}}| \rangle_{\alpha \in [0, \alpha_{\text{max}}]} \quad (13)$$

together with their standard deviations provide an α dependent error for a scaling index analysis under the specified conditions. If deviations within an interval $[0, \alpha_{\text{max}}]$ are significantly larger than the corresponding value of $\langle \Delta(\alpha_{\text{max}}) \rangle$ (together with its standard error), then the deviation is significant with respect to the fluctuations of a properly selected random distribution.

Since we are ultimately interested in deviations from randomness in data taken under experimental conditions as indicated in Sec. 1.2, another (maybe even more) proper candidate for a random reference are the calibration data themselves rather than a Gaussian distribution fitted to them. Subtracting the mean histogram $\langle N(\alpha)_{\text{cal}} \rangle$ from each individual histogram of the 100 different sets of calibration data, the resulting integrated normalized differences are:

$$\Delta_{\text{cal}} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (N(\alpha)_{\text{cal},i} - \langle N(\alpha)_{\text{cal}} \rangle) d\alpha. \quad (14)$$

As for the Gaussian distributions, one can now determine the mean values $\langle \Delta(\alpha_{\text{max}}) \rangle$ over all realizations i and for $\alpha \in [\alpha_{\text{max}}]$ according to:

$$\langle \Delta(\alpha_{\text{max}}) \rangle = \langle \max |\Delta_{\text{cal}}| \rangle_{\alpha \in [0, \alpha_{\text{max}}]}. \quad (15)$$

These mean values (solid line) and their standard deviations (shaded area) are plotted as a function of α_{\max} in Fig. 4. Its most characteristic feature is a steep increase of $\langle\Delta(\alpha_{\max})\rangle$, starting at about $\alpha = 3.0$ and reaching its maximum at about $\alpha = 3.3$. The corresponding numerical values of $\langle\Delta(\alpha_{\max})\rangle$ and their standard deviations $\sigma(\alpha_{\max})$ for $2.9 \leq \alpha_{\max} \leq 3.4$ are listed in Table 1. The overall mean of extrema is given by

$$\langle\max|\Delta_{\text{cal}}|\rangle_{\alpha} = (1.14 \pm 0.82)\%. \quad (16)$$

The behavior of $\langle\Delta_{\text{Gauss}}\rangle$ and $\langle\Delta_{\text{cal}}\rangle$ as a function of α_{\max} can be compared in Fig. 5.

3.2 Monte Carlo Permutations

Another valuable technique to check for errors in simulated random distributions is based on Monte Carlo techniques.³ Here one starts with a given time series (corresponding to a given distribution of state probabilities) and randomizes the sequence of states. This is to say that – by contrast to the generation of Gaussian distributions – only transition probabilities between states are randomized whereas the state probabilities themselves remain unchanged. Monte Carlo studies were carried out with respect to two different types of initial time series (defining the state distribution): (1) a mean Gaussian distribution and (2) a mean distribution of calibration data.

1. From the mean Gaussian distribution of states, all (10000) individual states were used to create a random sequence of states. This sequence can be interpreted as a random “time series” and serves as an initial time series for a total number of 100 Monte Carlo permutations. For each of these permutations, the differences of the corresponding $N(\alpha)$ histograms and the histogram for the mean Gaussian distribution were calculated, providing

$$\Delta_{\text{perm,Gauss}} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (N(\alpha)_{\text{perm},i} - \langle N(\alpha)_{\text{Gauss}} \rangle) d\alpha. \quad (17)$$

Moreover, one can determine a mean histogram $\langle N(\alpha)_{\text{perm}} \rangle$ and subtract this from the histogram of each individual permutation. The resulting integrated normalized differences are:

$$\Delta_{\text{perm,perm}} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (N(\alpha)_{\text{perm},i} - \langle N(\alpha)_{\text{perm}} \rangle) d\alpha. \quad (18)$$

From these two versions of $\Delta_{\text{perm},\dots}$, mean values $\langle\Delta(\alpha_{\max})\rangle$ over all realizations and for $\alpha \in [\alpha_{\max}]$ were calculated according to

$$\langle\Delta(\alpha_{\max})\rangle = \langle\max|\Delta_{\text{perm},\dots}|\rangle_{\alpha \in [0, \alpha_{\max}]}. \quad (19)$$

³Related procedures involving higher degrees of sophistication are provided by so-called surrogate data techniques (Theiler et al. 1992).

The dash-dotted and long-dashed curves in Figure 5 show these mean values as a function of α_{\max} .

The curves of $\langle \Delta(\alpha_{\max}) \rangle$ for $\Delta_{\text{perm,Gauss}}$ and $\Delta_{\text{perm,perm}}$ coincide remarkably well, indicating that the mean distributions $\langle N(\alpha)_{\text{Gauss}} \rangle \approx \langle N(\alpha)_{\text{perm}} \rangle$. Especially in the range $\alpha > 3.2$, the mean maxima of $|\Delta_{\text{Gauss}}|$ are considerably larger than the curves resulting from permutations. This result is due to the fact that individual different realizations of Gaussian distributions are broader than their mean distribution. Since the permutations are based on the *mean* state distribution, the resulting scatter is smaller than the scatter between individual Gaussian distributions.

2. As an alternative to individual Gaussian realizations, 100 different series of calibration data, each with $N_{\text{tot}} = 10000$, were taken as individual time series. Their mean distribution served as a basis for Monte Carlo permutations. Again using 100 permutations, the differences of the corresponding $N(\alpha)$ histograms and the histogram for the mean distribution of permuted calibration data were calculated according to

$$\Delta_{\text{permc,cal,permc}} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (N(\alpha)_{\text{permc,cal,i}} - \langle N(\alpha)_{\text{permc}} \rangle) d\alpha. \quad (20)$$

From this version of Δ , mean values $\langle \Delta(\alpha_{\max}) \rangle$ over all realizations and for $\alpha \in [0, \alpha_{\max}]$ were determined according to:

$$\langle \Delta(\alpha_{\max}) \rangle = \langle \max_{\alpha \in [0, \alpha_{\max}]} |\Delta_{\text{permc,cal,permc}}| \rangle. \quad (21)$$

The dotted curve in Figure 5 shows these mean values as a function of α_{\max} . This curve coincides roughly with those for the other two Monte Carlo studies. For $\alpha < 3.2$, all five curves in Fig. 5 are almost identical. For $\alpha > 3.2$, there is a conspicuous difference between Monte Carlo studies and both Gaussian and calibration data. This difference is due to the fact that Monte Carlo permutations as performed here leave the state probabilities (of a mean distribution) unchanged, whereas different realizations of Gaussian or calibration distributions change the state probabilities, thus leading to larger differences between individual realizations.

The deviations between histograms $N(\alpha)$ and a suitable mean histogram $\langle N(\alpha) \rangle$ provide a measure of the size of (random) fluctuations in histograms $N(\alpha)$ characterizing a random distribution for $N_{\text{tot}} = 10000$. Therefore, $\langle \Delta(\alpha_{\max}) \rangle$ provides a suitable α sensitive error estimate for a scaling index analysis. If deviations within an interval $[0, \alpha_{\max}]$ are significantly larger than the corresponding value of $\langle \Delta(\alpha_{\max}) \rangle$, then the deviation is significant with respect to the fluctuations of a properly selected random distribution. For our experimental data, the most appropriate candidate for a random reference are the calibration data themselves rather than a fitted Gaussian distribution or any permutations. (As discussed above, permutation studies underestimate errors.) For this reason, $\langle \Delta(\alpha_{\max}) \rangle$ with Δ_{cal} (Eqs. 14,15, Tab. 1, Fig. 4, solid line in Fig. 5) has been selected as the most suitable error estimate.

For $\langle \max_{\alpha \in [0, \alpha_{\max}]} |\Delta_{\text{cal}}| \rangle$ and $\alpha_{\max} \in [0, 3.1]$, $\langle \Delta(\alpha_{\max} = 3.1) \rangle \approx 0.6\%$ can be obtained from Table 1. A deviation of the order of 2% at $\alpha = 3.1$ would be significant

at the 3σ level ($p \approx 0.004$, one-tailed) if considered with respect to an error given by $\langle\Delta(\alpha_{\max})\rangle$. By contrast, the same size of deviation at $\alpha = 3.4$ (where the error is close to 1.3) would only provide about 1.54σ , corresponding to a p -value of about 0.12. A global error estimate as given by Eq. (16) provides a significance level comparable to the α sensitive error for $\alpha = 3.2$.

Using the difference between an observed deviation and $\langle\Delta(\alpha_{\max})\rangle$, normalized by the standard error of $\langle\Delta(\alpha_{\max})\rangle$ according to Tab. 1, one obtains 3.17σ as a significance estimate for a deviation of 2% at $\alpha = 3.1$. While the first kind of error estimation (mentioned above, i.e., with respect to $\langle\Delta(\alpha_{\max})\rangle$) provides the deviation of an observed distribution in units of a reference distribution, the second provides the deviation of an observed distribution in units of the standard error of the reference distribution (i.e., with respect to the standard deviation of $\langle\Delta(\alpha_{\max})\rangle$). The following analysis concentrates on deviations and their significances as determined according to the first procedure.

It should be emphasized that a significance estimate based on the extrema of differences in each α range rather than those differences themselves is a conservative estimate. First, $\langle\Delta(\alpha_{\max})\rangle$ represents a supremum for an error based on the average over all differences in each α bin individually. Both error estimates coincide roughly for small α , i.e., at the left wing of the $N(\alpha)$ histogram where detectable deviations due to non-random contributions are to be expected. Second, if one considers the integral differences for individual data sets not only at their extrema but also for smaller values of α , the α sensitive significance increases considerably in some cases (e.g., in the example of Fig. 1). This effect will be taken into account in more detail in the following section.

A final remark concerning the interpretation of deviations from randomness in terms of correlations: values of Δ_{cal} that differ from zero quantify how much the density of an individual set of calibration data fluctuates around the density of an average distribution of calibration data with $N_{\text{tot}} = 10000$ on purely statistical grounds (given that the overall density is constant). Deviations in the left wing of $N(\alpha)$ refer to “overdense” regions in the embedding space. They indicate that local correlations in an individual set of calibration data deviate from those of an average distribution of calibration data, i.e., are more or less homogeneous than that average distribution (with a finite number of points). “Underdense” regions such as voids (characterized by “anti”-correlations) correspond to the right wing of $N(\alpha)$ where errors typically are much larger and prevent any significant detection (cf. Atmanspacher et al. 1989).

In summary, a fairly conservative and most suitable error estimate for the analysis is given by $\langle\Delta(\alpha_{\max})\rangle$ for the extrema of the differences Δ_{cal} (Eqs. 14,15) between the $N(\alpha)$ histograms of individual calibration data and their mean distribution. Numerical values and their standard deviation are listed in Table 1 and plotted in Fig. 4. The overall error (over the entire α range) is given by Eq. (16). In Figure 5, the α sensitive error due to Δ_{cal} (solid line) can be compared with other possible error estimates.

4 Data Analysis and Results

The empirical material that will be analyzed in this section is taken from a series of experiments carried out by three of the authors (EB, HB, RDN) at the Institut für Grenzgebiete der Psychologie und Psychohygiene (IGPP) at Freiburg/Germany. The purpose of this study, which is not yet (December 1997) finished, is to replicate the results obtained by PEAR as briefly summarized in Sec. 1.2. The protocol for the replication study requires that any pre-analysis has to remain blind with respect to correlations between individual agents and their results as well as with respect to the actual modes of intention (high, low, baseline). Therefore, individual agents were coded by numbers (1, ..., 16), and modes of intention were randomly coded by numbers (1, 2, 3) for each agent separately before the data were released for analysis. Hence the analysis, carried out by the other two authors (HA, HS) at the Max-Planck-Institut für extraterrestrische Physik (MPE), amounts to nothing else than an analysis of deviations from randomness. In particular, it does not make use of any information concerning correlations with psychological observables.

Only those agents were selected for the analysis who had finished 10 sessions within the IGPP replication study by the end of September 1997. Due to the experimental protocol, the corresponding amount of data is 10000 per mode of intention and per agent. However, these data were partly collected under different conditions. One of these differences is in the length of individual runs in a session. For some of the agents (# 1, 4, 11, 12) all sessions consisted of 30 runs each, 10 for each mode of intention, where each run consists of 100 data points (trials). One agent (# 9) did runs with 1000 data points exclusively, i.e., three runs (one for each intentional mode) per session. Most agents chose to mix runs of 100 with runs of 1000 data points (trials) in different sessions. Other differences concerning experimental conditions consist of volitional or instrumental data sampling and different kinds of feedback (compare Sec. 1.2).

For the scaling index analysis to be presented in the following, we looked at time series provided by sequences of 10000 data points per intentional mode for each agent, regardless of any other possible discriminations. This is to say that we disregarded any information other than the distinction between different but unknown (not yet decoded) modes of intention. In other words: we assume that each sequence of 10000 data points represents a statistical ensemble. The reason is that pilot studies showed that the deviations from randomness which can be expected are so small that a sensible scaling index analysis requires data series of at least 10000 points to be analyzed in at least four dimensions, $d = 4$. (Other parameters are as given in the beginning of Sec. 3.) More detailed studies (with shorter time series) may be possible if agents can be identified for which major deviations from randomness are observed.

The first step of the analysis was the determination of the relative integral differences between the $N(\alpha)$ histogram for a suitable representative for a random sequence and the three $N(\alpha)$ histograms corresponding to the three modes of inten-

tion for each agent. Selecting the mean histogram over all calibration data (cf. Sec. 3) as such a suitable representative, one has to calculate

$$\Delta_{\text{cal,int}_j} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (\langle N(\alpha)_{\text{cal}} \rangle - N(\alpha)_{\text{int},j}) d\alpha \quad (22)$$

for each agent. The solid line in Fig. 1 represents $\Delta_{\text{cal,int}_2}$ as a function of α for intentional mode #2 of agent #16. There is a prominent negative peak of 3.56% at $\alpha = 3.18$, where the corresponding error is 1.11 (see Tab. 1). This means that the deviation from randomness in this example is significant with 3.2σ . Among the agents investigated, this deviation (corresponding to a p -value of roughly 0.0017) is the most pronounced effect resulting from an analysis of the extrema of $\Delta_{\text{cal,int}_j}$. Its significance would remain essentially unchanged if the error were not determined in an α sensitive manner but with respect to the overall mean extremal deviation of $\Delta_{\text{cal}} = 1.14$ (for $\alpha_{\text{max}} \rightarrow \infty$).

The fact that this as well as all other significant deviations for all 16 agents are located at $\alpha < 3.3$ confirms the expectation that detectable deviations from randomness are in the left wing of the histograms. Columns 2–4 in Table 2 show the maxima and minima of $\Delta_{\text{cal,int}_j}$ in the range of $\alpha < 3.3$. Bold numbers indicate deviations that are more than 2σ significant with respect to the α sensitive error as given in Tab. 1. Subtracting one hidden set of calibration data, 6 out of 45 data sets are 2σ significant. Two of them belong to agents #16 and #11, one to #9 and #3.

Some of the 2σ significant peak deviations are positive, others are negative. Positive deviations indicate that data taken under agent intention lead to $N(\alpha)$ histograms whose left wing is less populated than that of the random reference histogram. This means that the sequence under agent intention is more homogeneous than the random reference sequence with $N_{\text{tot}} = 10000$. Negative deviations indicate that data taken under agent intention lead to $N(\alpha)$ histograms whose left wing is more populated than that of the random reference histogram. This means that the sequence under agent intention is less homogeneous than the random reference sequence, i.e. contains correlations that are absent in the random reference series. It should be added that both statements apply with respect to the selected locality criterion (see Sec. 3). It will be instructive to further explore these deviations when the assignment to actual modes of intention and the “success” of the agent can be decoded.

As mentioned in Sec. 3, an analysis of the significances of differences Δ might lead to underestimations if one considers only the extrema of Δ . This is due to the fact that the errors in the critical range $2.9 < \alpha < 3.3$ depend strongly on α . As a consequence, deviations smaller than their extrema may be more significant than those at the extrema, particularly if they are located at small enough values of α . For this reason, it is interesting to check the dependence of significances of deviations on α in addition to the deviations themselves. Figure 6 shows a corresponding plot for those agents/intentions (7/3, 8/2, 11/2, 16/2) showing deviations more

significant than 2.5σ for $\alpha < 3.3$.⁴ For instance, the significance of deviations for agent #16/intention #2 turns out to be highest at $\alpha = 2.98$ where an overwhelming 9.52σ is obtained. Note that there are no positive deviations left for a significance threshold of 2.5σ .

Since the analysis as performed in this study is sensitive to deviations from randomness in the transition probabilities as well as state probabilities, it is desirable to distinguish between the two different types of non-randomness in the experimental data. If the deviations are due to non-random transition probabilities, they should vanish (or at least decrease) for a Monte Carlo randomization of the sequence of states. This can be tested by generating random permutations of the time series taken under agent intention and subtracting the resulting $N(\alpha)$ histogram from the mean histogram of the calibration data. For a first check, only five permutations have been used for the data taken under agent intention. The relative integral differences

$$\Delta_{\text{cal,intperm}_j} = \frac{1}{N_{\text{tot}}} \int_0^{\alpha'} (\langle N(\alpha)_{\text{cal}} \rangle - N(\alpha)_{\text{intperm}_j}) d\alpha \quad (23)$$

as a function of α can then be compared with the relative integral differences $\Delta_{\text{cal,int}_j}$ according to Eq. (22). It is again particularly interesting to focus on the left wing of the histograms, i.e., at $\alpha < 3.3$. It turns out that all significant deviations (cf. Tab. 2 and Fig. 6) drop well below 2σ if the corresponding sequences are randomized.

The hypothesis that we are dealing with deviations from randomness in the transition probabilities rather than randomness with respect to states can be further backed up by a simulation of small temporal correlations within an otherwise random time series. As an example, one can replace a small proportion, say 60 randomly selected sets of three successive data points (“triplets”) in a random series of calibration data with $N_{\text{tot}} = 10000$ by 60 ordered triplets of the form $\{91, 93, 95\}$. Analysing the modified time series according to the procedure described above, leads to deviations Δ from the mean calibration distribution which are plotted as a solid line in Figure 7. The dotted line in the same figure reproduces (from Fig. 1) the deviations obtained for agent #16/intention #2.

5 Summary

The general objective of the investigations described in this paper is the detection of deviations from randomness for empirical collectives of events under different experimental conditions. To detect these deviations, a scaling index analysis has been applied that combines established procedures for time series reconstruction and pattern detection. It is capable of discriminating faint non-random contributions to

⁴A more detailed investigation would have to take into account errors over the entire range of α for each α_{max} separately. In general, individual distributions of errors over α are different for different α_{max} . Figure 6 presupposes that the error at any α is given by the error for $\alpha = \alpha_{\text{max}}$ as in Tab. 1 (Fig. 4).

an otherwise random distribution for a finite number of events, i.e., independently of any limit theorems. Moreover, it allows us to check for non-random features in the state probabilities as well as non-random features in the probabilities of transitions between states.

To identify any deviations from randomness, a significance level has to be specified. For this purpose, a detailed error analysis has been carried out based upon random calibration data, simulated Gaussian data, and various Monte Carlo type studies of permutations of those data. The mean difference between individual distributions and mean distributions for each type of data has been used to define proper error estimates. Deviations exceeding this error significantly provide evidence for non-random contributions within an otherwise random distribution of data.

The analyzed data have been generated by a physical random event generator. Time series taken from this device without additional constraints have been used as random calibration data. Under experimental conditions in which human agents are asked to intentionally try to act (according to a specified protocol) such as to effectively introduce non-random contributions to the random calibration data, significant deviations ($> 2.5\sigma$) are obtained for some of the agents.

These deviations disappear if the temporal sequence of events is randomized, suggesting that the observed non-random contributions are due to temporal correlations in the data. Considering the analyzed data as statistical ensembles, it can be inferred that the origin of those deviations lies in non-random features in the distribution of transitions between states rather than the distribution of states themselves. Analyzing a random calibration sequence with a small proportion of randomly implemented ordered (correlated) triplets leads to deviations that are comparable to those under human agent intention and offers confirmation for this suggestion.

Our results do strongly encourage further investigations of deviations from randomness in physically random devices and their relationship to the intention of human agents with appropriate techniques. In particular, it seems worthwhile to focus on the *dynamical* aspects of such deviations in addition to more straightforward analyses of state probabilities.

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7 Figure and Table Captions

Figure 1: Example of differential and integral representations of the differences between two $N(\alpha)$ histograms. Dotted line: relative normalized differences

$\delta_{1,2}$ for each bin. Solid line: relative normalized integral differences $\Delta_{1,2}$.

Figure 2: Relative differences (in %) between the mean state distribution of calibration data and the mean of 100 realizations of a Gaussian state distribution with the same FWHM.

Figure 3: Extrema of Δ_{Gauss} for 100 realizations of a Gaussian state distribution as a function of α (in steps of $\Delta\alpha = 0.05$).

Figure 4: Mean values (solid line) and standard deviations (shaded area) of $\Delta(\alpha_{\text{max}})$ for the maxima of $|\Delta_{\text{cal}}|$ as a function of α_{max} .

Figure 5: Mean values of $\Delta(\alpha_{\text{max}})$ as a function of α_{max} . Short-dashed line: $\Delta = \Delta_{\text{Gauss}}$ (Eq. 11); solid line: $\Delta = \Delta_{\text{cal}}$ (Eq. 14) as in Fig. 4; dash-dotted line: $\Delta = \Delta_{\text{perm,Gauss}}$ (Eq. 17); long-dashed line: $\Delta = \Delta_{\text{perm,perm}}$ (Eq. 18); dotted line: $\Delta = \Delta_{\text{permc,permc}}$ (Eq. 20). Errors of Δ_{cal} as in Figure 4 are comparable to any other errors.

Figure 6: Significance level of deviations (in units of σ as given by $\langle\Delta_{\text{cal}}\rangle$) as a function of α for those agents/intentions (7/3, 8/2, 11/2, 16/2) with significances higher than 2.5σ for $\alpha < 3.3$. Note that all significant deviations are at values of α which are smaller than those α at which the deviations themselves are extremal. Note also that all 2.5σ significant deviations are negative.

Figure 7: Solid line: Relative normalized integral differences Δ between the $N(\alpha)$ histograms of a distribution with 60 ordered triplets and a mean random calibration distribution. Dotted line: Relative normalized integral differences Δ between the $N(\alpha)$ histograms of the distribution corresponding to agent 16/intention 2 and a mean random calibration distribution.

Table 1: Numerical values of $\langle\Delta(\alpha_{\text{max}})\rangle$ for Δ_{cal} and its error $\sigma(\alpha_{\text{max}})$ for $2.9 \leq \alpha_{\text{max}} \leq 3.4$.

Table 2: Overview of results of the scaling index analysis for experimental data involving human agent intention. Modes of intention are coded by $j = 1, 2, 3$. First column: # of agent. Columns 2–4: maxima/minima of $\Delta_{\text{cal,int}_j}$ for $\alpha < 3.3$. Bold numbers in columns 2–4 are deviations above 2σ , considered with respect to the α sensitive errors as given in Tab. 1.

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