# Some Basic Problems With Complex Systems

Harald Atmanspacher[1,2] and Gerda Wiedenmann[2]

1 Institut für Grenzgebiete der Psychologie,
D–79098 Freiburg
2 Max-Planck-Institut für extraterrestrische Physik,
D–85740 Garching

## Abstract

From an engineering perspective, it is well known that there are numerous problems to predict and control complex systems. In addition, there are also problems to understand the concept of complexity from the perspective of physics and the epistemology of physics. Three outstanding topics in this regard are discussed: 1. the fundamental *context-dependence* of the definition of complexity, 2. the relation between complexity and *meaning*, and 3. the restrictions on the applicability of *limit theorems* in the study of complex systems.

## 1 Introduction

The concept of complexity and the study of complex systems represent an important focus of research in contemporary science. Although one might say that its formal core lies in mathematics and physics, complexity in a broad sense is certainly one of the most interdisciplinary issues scientists of almost any conceivable background talk about today. Beyond the traditional disciplines of the natural sciences, the "virus" of complexity has even crossed the border to areas like psychology, sociology, ecology and others. It is entirely impossible to address all approaches and applications that are presently known comprehensively here; good overviews including state-of-the-art articles as well as more tentative ideas have been published by Cowan et al. (1994) and Cohen and Stewart (1994).

From the viewpoint of modern physics, the study of complex systems can be understood as a continuation of a whole chain of interdisciplinary approaches, leading from system theory (Bertalanffy 1968) and cybernetics (Wiener 1948) to synergetics (Haken 1977) and self-organization (Foerster 1962), dissipative (Nicolis and Prigogine 1977) and autopoetic systems (Maturana and Varela 1980), automata theory (Hopcroft and Ullmann 1979), and others. In all these approaches, the concept of information plays a significant role in one or another way, first due to Shannon and Weaver (1949) and later also in other contexts (Zurek 1989, Atmanspacher and Scheingraber 1990, Kornwachs and Jacoby 1996, Marijuàn and Conrad 1996).

A most important recent predecessor of complexity is the theory of nonlinear dynamical systems, which originated from early work of Poincaré and was further developed by Lyapunov, Hopf, Krylov, Kolmogorov, Smale, Ruelle – to mention just a few names. Prominent areas in the study of complex systems as far as it has evolved from nonlinear dynamics are fractals (Mandelbrot 1983), chaos (Stewart 1990), cellular automata (Wolfram 1986), and coupled map lattices (Kaneko 1993). Another trendy field in current complexity research is self-organized criticality (Bak and Chen 1991).

This ample list notwithstanding, it is fair to say that one important *open* question is the question for a fundamental theory, e.g., in the sense of an axiomatic basis, of nonlinear dynamical systems. Although much progress has been achieved in understanding a large corpus of phenomenological features of dynamical systems, we do not have any compact set of basic equations (like Newton's, Maxwell's, or Schrödinger's equations), or postulates (like those of relativity theory) for a comprehensive, full-fledged, formal theory of nonlinear dynamical systems. The same point can certainly be made with respect to the concept of complexity.

Which criteria does a system have to satisfy in order to be complex? This question is not yet answered comprehensively, too, but quite a few essential points can be indicated. From a physical point of view, it is often assumed that a situation far from thermal equilibrium is necessary for the emergence of complexity. This is to say that one usually does not speak of a complex system if its behavior can be described by the laws of linear thermodynamics. (In fact the entire terminology of equilibrium thermodynamics may become inapplicable in such situations.) The thermodynamical branch of a system has to become unstable before complex behavior can emerge. In this manner the concept of instability becomes an indispensable element of any proper understanding of complex systems. This can mean that the entire In addition, complex systems are usually regarded as open systems, exchanging energy and/or matter (and/or information) with their environment. Other features which are most often found in complex systems are internal self-reference (e.g., feedback) and external boundary conditions such as control parameters (e.g., energy/matter inflow). Sometimes it is argued that external boundary conditions may gradually become internalized, i.e., become part of the internal dynamics of a system, if one is dealing with living biological systems (see, e.g., Atlan 1990). In this context, Maturana and Varela (1980) have established the concept of autopoiesis accounting for the fact that living system are able to develop (and modify) their own boundaries. Details about what might happen at higher levels in the hierarchy, for instance at the levels of cognitive or even social systems, are basically unclear at present.

## 2 Definitions of Complexity

Problems with standard scientific methodology arise already in the realm of definitions. Subsequent to algorithmic complexity measures like those of Solomonoff (1964), Kolmogorov (1965), Chaitin (1966), and Martin-Löf (1966) (see also Schnorr 1971), a huge number of definitions of complexity have been suggested (for overviews, see Lindgren and Nordahl 1988, Grassberger 1989, or Wackerbauer et al. 1994). Though some of them seem to be more popular than others, there exist no clear or rigorous criteria to select a "correct" definition and reject the rest. More and more it appears that one of the basic principles of scientific methodology, the principle of *universality*, cannot be maintained naively, but has to be complemented by an unavoidable context-dependence, or *contextuality*. Important examples for such contexts are the role of the environment, resembling the corresponding distinction for the measurement problem in modern quantum theory (Atmanspacher et al. 1995), and the role of the model class an observer has in mind when he tries to model a complex system (Crutchfield 1994). For a more detailed account of some epistemological background to these topics compare Atmanspacher (1997).

A systematic orientation in the jungle of concepts of complexity is impossible unless a reasonable classification is at hand. As with the definitions there are again several approaches that can be found in the literature: two of them are (i) the distinction of structural and dynamical measures (Wackerbauer et al. 1994) and (ii) the distinction of deterministic and statistical measures (Crutchfield and Young 1989; note that deterministic measures are not non-statistical in the sense that they avoid statistical tools of quantification). Another, epistemologically inspired (Scheibe 1973) scheme (iii) assigns ontic and epistemic levels of description to deterministic and statistical measures, respectively (Atmanspacher 1994). In addition to these approaches a purely phenomenological criterion for classification is given by the way in which a complexity measure is related to measures of randomness. Before entering this issue it should, however, be emphasized that randomness itself is a concept that is anything else than finally clarified. In the framework of the present paper we use the notion of randomness in the broad sense of an entropy. As far as dynamical randomness is concerned, we refer to a variety of different random processes as discussed by Gaspard and Wang (1993), see also Deco et al. (1997).

From a point of view that contrasts complexity and randomness (for an early reference in this regard, see Weaver 1968), there are two classes of complexity measures: (iv) those for which complexity increases monotonically with randomness and those with a globally convex behavior as a function of randomness. It turns out that classifications according to (ii) and (iii) distinguish measures of complexity precisely in the same manner as (iv) does: all deterministic or ontic measures behave monotonically, and all statistical or epistemic measures are convex. In other words: deterministic (ontic) measures are essentially measures of randomness, whereas statistical (epistemic) measures are not. The first class contains, e.g., algorithmic complexity (Kolmogorov 1965), various kinds of Rényi information (Balatoni and Rényi 1956), among them Shannon's information (Shannon 1949), multifractal scaling indices (Halsey et al. 1986), or dynamical entropies (Kolmogorov 1958). The second class contains, e.g., effective measure complexity (Grassberger 1986), $\epsilon$-machine complexity (Crutchfield and Young 1989), fluctuation complexity (Bates and Shepard 1991), variance complexity (Atmanspacher et al. 1997). See also Landsberg and Shiner (1998) and Feldman and Crutchfield (1998).

A most intriguing further difference (v) between both classes can be recognized if one focuses on the way statistics is implemented in each of these measures. The crucial point is that convex measures, in contrast to monotonic measures, are *meta*-statistically formalized, i.e., effectively represent (in one or another way) second-order statistics in the sense of "statistics of statistics".[1] Fluctuation complexity is the standard deviation (second-order) of a net mean information flow (first-order), effective measure complexity is the

[1]Note that the notion of "second-order statistics" has nothing to do with the second moment of a statistical distribution.

convergence rate (second-order) of a difference of entropies (first-order), $\epsilon$-machine complexity is the Shannon information with respect to machine states (second-order) that are constructed as a compressed description of a data stream (first-order), and variance complexity is based on the variance (second-order) of the mean of many individual variances of a distribution of data. To our knowledge, there is no monotonic complexity measure providing such a two-level statistical structure. Clearly, it would be desirable to have a theorem for the corresponding relationship between convex complexity measures and their two-level statistical structure. Such a theorem is not yet available though.

# 3 Meaning

To our knowledge, Grassberger (1986) and Atlan (1987) were virtually the first to emphasize a close relationship between complexity and meaning. In Grassberger's words (Grassberger 1989, his italics), "complexity in a very broad sense is a *difficulty* of a *meaningful task*. More precisely, the complexity of a pattern, a machine, an algorithm etc. is the difficulty of the most important task related to it. ... As a consequence of our insistence on *meaningful* tasks, the concept of complexity becomes *subjective*. We really cannot speak of the complexity of a pattern without reference to the observer. ... A unique definition (of complexity) with a universal range of applications does not exist. Indeed, one of the most obvious properties of a complex object is that there is no *unique* most important task related to it." Although this remarkable statement was made about ten years ago, it is still quite unclear how the relation between complexity and meaning looks in detail. Before we come to this, some words about the notion of meaning in general are necessary.

Originally, philosophers alone have been interested in the notion of meaning. In the last century, Schleiermacher and Dilthey laid the foundations of what is today known as the "hermeneutic method" in philosophy. Another approach to the notion of meaning has later been introduced by Charles Sanders Peirce and, more specifically, Morris (1955): the semiotic approach. In simple terms, semiotics is the study of signs. It is constituted by three different fields, namely syntactics, semantics, and pragmatics. While the syntactic level is relevant for the interrelation of signs, semantics deals with the relation between signs and what they designate, and pragmatics focuses on the relation between signs and their users. Applying this semiotic scheme to the realm of scientific models, one can distinguish between the syntactic level of the pure formalism of a model, the semantic level of its interpretation, and the pragmatic level of its application. Semantics addresses the meaning of the formalism, and pragmatics addresses its usage.

However, this clear distinction is itself justified only at an abstract level which is purely syntactical. As soon as one starts to consider aspects of constructing, testing, and working with a model concretely, any rigorous demarcation dissolves. This has the ultimate consequence that syntactics, semantics, and pragmatics lose their separate ranges of applicability (for a more detailed discussion see Atmanspacher 1994). Nevertheless, the separation of these ranges maintains its relevance on a conceptual level, and it is certainly an immensely helpful tool at this level. As long as one keeps the restriction to abstraction consciously in mind, it is thus justifiable to talk about syntactics, semantics, and pragmatics as separable and separate fields. Within the present context, this argument allows one to talk about meaning as the central notion of semantics without explicitly incorporating syntactics and pragmatics at the same time.

The variety of different complexity measures indicates that information theoretical approaches are promising in this respect; they are an integral part of most approaches to define complexity in the framework of computational physics. Of course, the way information theory is used in physics up to now is limited to its syntactical component. It goes back to the influential work Shannon published in a book authored together with Weaver (Shannon and Weaver 1949). Shannon-type information is purely syntactical in omitting any reference to contextual or meaning-related issues. It vanishes in case of a message that leaves the receiver in a given state of "syntactic" ignorance, i.e., the state of "syntactic" knowledge remains unchanged. It is positive, if the amount of ignorance of the receiver is reduced.

Weaver's contribution in the mentioned book (Shannon and Weaver 1949) expressed already that this syntactical component of information requires extension into semantic and pragmatic aspects. Shortly later, Bar Hillel and Carnap (1953) proposed a quantification of semantic information based on a receiver's ability to draw logical conclusions from received messages. The basic question here is: What happens if a message contains a huge amount of syntactic information which is not or cannot be understood by its receiver? For instance, imagine a "Babylonian library", containing all books which can ever be written. Obviously, most such books are meaningless to most (standard) readers because the sequences they offer merely satisfy a prescribed set of syntactic rules (if at all). The library thus contains an incredible amount of syntactic information, but only a small fraction of it (depending on the reader) is semantic information.

How is it possible to check if a receiver has understood the meaning of a message? A provisional answer is that semantic information can be qualified by its usage. If meaning is understood, then it triggers action or leaves some other imprint on the structure or behavior of the receiver. But still the problem remains unsolved how to judge the response of the receiver, if it responds in an unknown language or, even worse, if its response is non-verbal or totally untransparent. Is there any critical point in the hierarchy of complex systems, beyond which meaning is a meaningful issue and below which it is not (see, e.g., Atlan 1991)?

As a starting point toward an improved understanding of these problems, E. von Weizsäcker (1974) has introduced a way to deal with the usage that is based on the meaning of a message in terms of pragmatic information. This concept is based on the two notions of *primordiality* ("Erstmaligkeit") and *confirmation* ("Bestätigung"). Weizsäcker argued that a message that does nothing but confirm the prior knowledge of a receiver will not change its structure or behavior. On the other hand, a message providing material completely unrelated (primordial) to any prior knowledge of the receiver will not change its structure or behavior, simply since it will not be understood. In both cases, the pragmatic information of the message vanishes. A maximum of pragmatic information is assigned to a message that transfers an optimum mixture of primordiality and confirmation to its receiver. It is interesting to note that (purely syntactic) Shannon information represents a limiting case of pragmatic information for complete confirmation. If primordiality is added it increases monotonically.

The concept of pragmatic information can be made operationally accessible. Pioneering work in this respect is due to Gernert (1985) and to Kornwachs and Lucadou (1982, 1985) who applied pragmatic information to the study of cognitive systems. But also purely physical systems allow (though not require) a description in terms of pragmatic information. This has been shown in detail by Atmanspacher and Scheingraber (1990): particular events (instabilities) in a specific system (a laser system) can be considered as meaningful in the sense of positive pragmatic information if and only if they are accompanied by a change of the degree of complexity of the system. However, it is essential to realize that the concept of pragmatic information does not provide any surplus in explanative power in addition to a purely physical model of these systems. The reason is that the systems considered are *physical* systems without any reference to *mental* phenomena. Only if an explicit account of mental phenomena becomes unavoidable, an explanative surplus of pragmatic information can be expected. Applications in cognitive systems along the lines of Kornwachs and Lucadou (1982, 1985) seem to be most promising in this direction.[2]

Based on the concept of pragmatic information, an important conceptual connection between meaning and complexity can be established straightforwardly (cf. Atmanspacher 1994). Applying a proper algorithm in order to generate a regular pattern, e.g., a chess-board like period-2 pattern, the corresponding generation process is obviously recurrent after the first two steps, i.e., after the generation of the first two elements of the pattern. Considering the entire generation process as a process of information transmission, it is clear that any part of the process after its first two time steps represents but confirmation of these first steps. In this sense, a regular pattern of vanishing complexity corresponds to a process of information transmission that has vanishing meaning as soon as an initial transient phase (the first two time steps) has passed by.[3] This argument holds for both notions of complexity, the deterministic as well as the statistical one.

For a completely random pattern the situation is more involved, since deterministic complexity and statistical complexity lead to different viewpoints. Deterministically, a random pattern is generated by an incompressible algorithm which contains as many steps as the pattern contains elements. The process of generating the pattern is not recurrent within the length of the algorithm. This means that it never ceases to produce elements that are unpredictable, except under the assumption that the entire algorithm were *a priori* known. Such knowledge, however, would imply that the pattern itself were known, since the algorithm is nothing but an incompressible description of it. Hence, the process generating a random pattern can be interpreted as a transmission of information completely lacking confirmation, and consequently with vanishing meaning.

If the statistical notion of complexity is focused on, the process of pattern generation must no longer be considered as a process of generating the pattern out of individually distinct elements. A statistical generation of the pattern is not unique with respect to its local properties; it is only unique with respect to the global statistical distribution. Thus there is a significant shift in

[2] In cognitive science, there are many ways to deal with meaning other than in terms of pragmatic information. Among them, the process of learning is crucial for the study of semantic issues. Some background information on its relation to unstable cognitive processes can be found in Atmanspacher (1992). A recommendable article emphasizing the role of context in learning has been published by Bernasconi and Gustafson (1998).

[3] Of course, broader contexts are conceivable in which the same pattern remains "meaningful". For instance, the meaning of a stop sign is as significant as it causes us to stop in a specific traffic situation, although the sign *as such* has vanishing pragmatic information for those who have seen it often enough.

perspective which does not allow one to argue the same way as in the deterministic situation. While deterministic complexity refers to an individual, deterministic level ultimately lacking operational access, such access is always achievable by proceeding to a statistical level. It is suggestive to consider the issue of meaning exactly in the reverse sense. This is to say that the operationally accessible level for meaning is the individual, ontic level, whereas the statistical, epistemic level might at least be regarded as unoperational as far as meaning is concerned. A formal indication for the relevance of this suggestion is the fact that the convexity of statistical complexity coincides with the convexity of pragmatic information as a measure of meaning in the deterministic sense. It is remarkable how the perspectives of physics (complexity) and of cognitive science (meaning) show an amazingly explicit complementary feature in this respect.

# 4 Large Deviations, Limit Theorems, and Ergodicity

As another example for basic complications additional to those met in conventional physics, the study of complex systems includes non-stationary, transient states. Hence, ergodicity cannot be presupposed in general, and ergodic measures cannot be used without caution (Tanaka and Aizawa 1993). Moreover, it is now well known that careless applications of limit theorems in statistical analyses of data from complex systems can lead to pitfalls and misinterpretations (Wolfram 1984). These complications are only begun to be understood in detail. Apart from Crutchfield's highly developed approach in terms of $\epsilon$-machine reconstruction, the framework of large deviations statistics (Ellis 1985, Aizawa 1989, Oono 1989, Bucklew 1990, Seppäläinen 1995) offers itself as a promising route of access.[4] Large deviations statistics is particularly attractive since it distinguishes explicitly between statistical (monotonic) and meta-statistical (convex) measures of complexity.

A basic element in large deviations statistics (LDS) is a switch of perspective from statistical moments of a distribution, e.g., expectation values, to the probability measure itself, e.g., moments of a distribution of distributions. Moments of a distribution provide first-order statistical characterizations of this distribution. They are defined in the limit of $N \to \infty$, i.e., in a "thermodynamic" limit, where $N$ can be the number of particles, of degrees of freedom, of subensembles, etc. The corresponding "law of large numbers" states that in the thermodynamic limit a distribution converges weakly to the unit point measure at the expectation value. LDS specifies that the *rate* of this convergence is exponential as a function of $N$ (Ellis 1985, Oono 1989).

If an observable is defined in the sense of an expectation value, then the relevant framework is that of a level-1 description. For instance, the formalism of multifractal measures (Halsey et al. 1986, Paladin and Vulpiani 1987) is based on a thermodynamic limit; hence it is a level-1 theory and uses only first-order statistical measures. A more restrictive limit theorem which (other than a law of large numbers) presupposes the existence of the second moment of a distribution is the "central limit theorem". It gives an estimate for the probability that the size of properly defined, i.e., normalized, fluctuations around the expectation value is of the order of $\sqrt{N}$.

If the thermodynamic limit as a precondition for a law of large numbers in the sense of a level-1 description cannot be presupposed, one can consider a higher level at which the observed empirical distribution functions themselves (not single variables) are treated as stochastic objects. Measures on such a higher level are meta-statistical measures; they characterize the fluctuations of the distribution functions as a function of $N$. Distributions in a purely structural (non-dynamical) sense give then rise to meta-statistical level-2 descriptions. A good example is the behavior of histograms of scaling indices for finite $N$ as a function of $N$ (which become multifractal measures for $N \to \infty$). For distributions covering structural as well as dynamical elements it can be reasonable to proceed to meta-statistical descriptions that are called level-3 descriptions in the terminology of LDS (Ellis 1985, Oono 1989). The objects of these descriptions are trajectories or histories instead of level-2 distributions.

A level-$(n-1)$ theory can in general be obtained from the corresponding level-$n$ theory ("contraction principle"; Ellis 1985, Oono 1989). For instance it is possible to infer the convergence rate toward an expectation value (assuming that it exists) from the convergence rate of its probability distribution. An analogous contraction principle does not in general apply to the *moments*. If the distribution function depends on time, averages over time and ensemble averages are not necessarily identical (cf. Feller 1971). If this difference is not explicitly taken into account pitfalls with respect to the validity of a law of large numbers are almost to be expected. Pikovsky and Kurths (1994) have recently clarified such a misunderstanding for a level-3 situation (see also Griniasty and Hakim 1994). They have shown that properly defined higher-order fluctuations do not violate a level-3 law of large numbers whereas such a law is irrelevant for fluctuations in a lower level descrip-

[4] Relationships between large deviations statistics and $\epsilon$-machine reconstruction have been indicated by Young and Crutchfield (1994).

tion.

In more detail, Pikovsky and Kurths demonstrated that stationarity and ergodicity must not be presupposed in complex systems such as coupled map lattices or, more specifically, globally coupled maps. This is particularly interesting in view of the fact that under respective conditions such systems can provide long-living transients – a field of research far from being explored comprehensively. Non-stationary and non-ergodic behavior in this sense is expected to play a significant role in cognitive systems (cf. Freeman 1994, Nozawa 1994). In addition, coupled map lattices offer an interesting perspective for non-hierarchical "control" insofar as the behavior at each site in the lattice crucially depends on its environment (consisting of neighboring sites). It is not simply determined by an externally adjusted (set of) parameter(s), which is the central idea in hierarchical approaches such as controlling chaos (Ott et al. 1990).

# 5 Conclusions

These arguments indicate an intimate relationship between the phenomenological classes of monotonic and convex complexity measures and the first-order and higher-order statistical structure of these measures, respectively. Moreover, the different classes of complexity measures have been related to the relevance or irrelevance, respectively, of limit theorems (laws of large numbers, central limit theorems) that can in a compact and general manner be formalized within the framework of large deviations statistics. First-order limit theorems become irrelevant in systems requiring higher-order statistics as covered by large deviations statistics. On the other hand, it is well known that first-order limit theorems as they are usually applied often lose their relevance for the statistical analysis of complex systems. The equivalence of convex complexity measures and higher-order large deviations statistics thus provides strong and general guidelines with respect to a proper formal way to study complex systems.

The significance of this approach is further supported by a number of important epistemological issues (for details see Atmanspacher 1994, Atmanspacher et al. 1995, Atmanspacher 1997). It is now commonly accepted that the complexity of a system cannot be uniquely characterized unless an observer's model of this system is *explicitly* taken into account (cf., for instance, Grassberger 1989). This implies that complexity is not a property of a system "out there" but rather a property of the relation between a system "out there" and the model by which it is interpreted by an observer. As a straightforward consequence any model of this relationship has (at least) to be a meta-model (see, e.g., Casti 1992). For a theory of complexity in this sense it is thus mandatory to be a second-order (meta-) theory. Its referents are not merely measured facts or data but *also* first-order models of these data *and* the relation between both, e.g. the process of model-building, or learning.

This requires a profound change of perspective as compared to conventional methodological principles in science. One of the most remarkable points in this regard is the altered situation with respect to the issue of operationalization in a metatheoretical framework. It is obviously inadequate to confirm or reject a meta-model simply by a "naive" observation of a "pure" fact. Instead, proper "experiments" have to include the relationship between data and models. In general, their analysis has to be a meta-analysis, and in general it has to be based on meta-statistics instead of conventional first-order statistics. In this context the concept of prediction which has already received a strong blow by the theory of nonlinear dynamical, particularly chaotic, systems will have to undergo even more drastic changes in a theory of complex systems.

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