Chaos and Complexity Letters
Volume 2, Number 2/3, pp. 139–150
ISSN 1555-3995
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STABILITY CONDITIONS IN CONTEXTUAL EMERGENCE

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Abstract

The concept of contextual emergence is proposed as a non-reductive, yet well-defined relation between different levels of description of physical and other systems. It is illustrated for the transition from statistical mechanics to thermodynamical properties such as temperature. Stability conditions are crucial for a rigorous implementation of contingent contexts that are required to understand temperature as an emergent property. It is proposed that such stability conditions are meaningful for contextual emergence beyond physics as well.

1. Introduction

A basic strategy for the scientific description of any system, physical or otherwise, is to specify its state and the properties associated with that state, and then introduce their evolution in terms of dynamical laws. This strategy presupposes that the boundary of a system can be defined with respect to its environment, although such definition is often seriously problematic. If a system can be defined reasonably, there is usually more than one possibility for specifying states and properties. The fact that states and properties can be formally and rigorously defined in fundamental physical theories, such as quantum mechanics, distinguishes the structure of such theories as particularly transparent.

The situation is different in physical theories which are not regarded as fundamental (such as thermodynamics), or in descriptive approaches beyond physics (such as chemistry, biology or psychology). For this reason, attempts have been made to relate descriptions of systems, which are not fundamental in the sense mentioned above, to descriptions which are fundamental in this sense. The usual (and often too simple) framework in which corresponding relations are typically formulated is that of a hierarchy of descriptions. In a hierarchical picture (which can be refined in terms of more complicated networks of descriptions) there are higher-level and lower-level descriptions. More fundamental theories are taken to refer to lower levels.

In such a simple framework, reduction and emergence are relations between different levels of descriptions of a system, its states and properties, or the (dynamical) laws characterizing their behavior. In the philosophical literature, the usual guiding idea behind reductionist approaches is to “reduce” higher-level features to lower-level features. By way of contrast, emergentist approaches emphasize the higher-level features by stressing the irreducibility of at least some of their aspects to lower levels. In this way, the emergence of features at higher levels is related to the emergence of novelty.

While reductionists would argue that both necessary and sufficient conditions for higher-level features are already embodied at the lower level, this is false in many of the more important examples (e.g. thermodynamics and statistical mechanics). An alternative kind of interlevel relation, contextual emergence, was recently proposed (Bishop and Atmanspacher, 2006) as a less rigid, more appropriate scheme, in which necessary but not sufficient conditions for higher-level features are provided by the lower-level description.

Stability conditions are crucial guiding principles for contextual emergence, which might be helpful for applications beyond physics as well. Specifically, one may think of relations between different levels of descriptions in brain physiology, where one of the key questions is how properties of neuronal assemblies (i.e. populations of neurons) are related to properties of individual neurons and synapses. However, one may also think of relations between such neurobiological levels of description and their mental correlates at cognitive or psychological levels of description. An interesting candidate for interlevel relations of the latter kind will be presented elsewhere (Atmanspacher and beim Graben, this issue).

Here we start with a brief introduction to the idea of contextual emergence and compare it with other kinds of interlevel relations in section 2. Section 3 outlines the general mathematical framework in which contextual emergence can be worked out for detailed examples. A particularly well-known example is presented in section 4, where we illustrate the formalism with details regarding the contextual emergence of temperature (and related thermodynamical properties) from a description in terms of statistical physics. The role of stability conditions for contextual emergence will be emphasized in section 5. Section 6 summarizes the basic arguments and results.

2. Reduction and Emergence

Reduction and emergence are used in a variety of senses in the literature. In general terms, both concepts express ways to achieve a better understanding of some feature of a system in terms of other features which are assumed to provide such understanding. For the sake of simplicity, reduction and emergence schemes are typically organized in a hierarchical manner, such that levels of description or levels of reality are related to each other. As mentioned above, an analysis in terms of hierarchical levels often oversimplifies the picture. In general, non-hierarchical frameworks (cf. Günther 1976–1980) including other notions such as those of domains of description or domains of reality might be more appropriate.

As indicated by the distinction between levels of description and levels of reality, there is a difference between epistemological and ontological frameworks for reduction and emergence. Broadly speaking, descriptive terms are subjects of epistemological discourse while elements of reality are subjects of ontological discourse. Both types of discourse are used in reductionist and emergentist approaches. The concept of reference establishes a connection between descriptive terms and described elements of reality (leaving aside difficult questions about reference itself).

In addition to the distinction between epistemological and ontological discourse, one should distinguish between different types of features which are to be related to others. There are three main categories of relations: theories/laws to other theories/laws, properties to other properties, and wholes to parts. Clearly, relations between theories/laws are predominantly epistemological. The relation between wholes and parts, on the other hand, is primarily conceived ontologically insofar as it emphasizes elements of reality rather than their description. In the literature on property relations, both epistemological and ontological frameworks can be found. Property relations are sometimes meant ontologically (i.e., regarding properties of elements of reality) and sometimes epistemologically (i.e., regarding descriptive terms referring to properties of elements of reality).

An ontological framework of discussion is usually employed in reductive approaches, where ontic elements are restricted to a fundamental level of description, at which those properties reside to which all other properties are regarded reducible and from which all other properties are regarded as exhaustively determined. An alternative idea of a tiered ontology, ascribing ontic elements to all levels of description, was proposed originally by Hartmann (1935). Quine (1969) has revitalized this idea with his notion of an ontological relativity. It was adopted by Putnam (1987) when he suggested his idea of internal realism, later denoted pragmatic realism. These philosophical frameworks of thinking were fleshed out by Atmanspacher and Kronz (1999) from a scientific perspective. This option presupposes a distinction between ontic and epistemic descriptions of the behavior of physical systems due to Scheibe (1973) and Primas (1990). A comprehensive review can be found in Atmanspacher and Primas (2003).

Analogous to Hartmann's and Quine's approaches, this allows us to conceive ontic elements at each level of description. In addition, however, it allows us to formally propose ways in which interlevel relations can be designed. In a nutshell, an ontic description at one level serves as the basis for an epistemic description at a higher level, which can be "ontologized" and then provides the basis for proceeding to another epistemic description at yet another level. (For details see Atmanspacher and Kronz 1999). If one wants to have the option of ontic elements at each level of description rather than only at a fundamental one, a straightforward and strictly reductive scheme for interlevel relations becomes impossible and must be relaxed. The way in which ontic and epistemic descriptions are related to each other motivates contextual emergence as a viable alternative.

In order to clearly distinguish between different concepts of reduction and emergence, it is desirable to have a transparent classification scheme, so that their basic characteristics can be discussed coherently. A useful approach toward such a classification is based on the role which contingent contexts play in reduction and emergence. More precisely, the way in which necessary and sufficient conditions are assumed in the relation between different levels of description can be used to distinguish four classes of relations:

(1) The description of features of a system at a particular level of description offers *both necessary and sufficient* conditions to rigorously derive the description of features at a higher level. This is the strictest possible form of *reduction*. It was most popular

under the influence of positivist thinking in the mid-20th century.

(2) The description of features of a system at a particular level of description offers *necessary but not sufficient* conditions to derive the description of features at a higher level. This version is called *contextual emergence*, because contingent contextual conditions are required in addition to the lower-level description for a rigorous derivation of higher-level features.

(3) The description of features of a system at a particular level of description offers *sufficient but not necessary* conditions to derive the description of features at a higher level. This version includes the idea that a lower-level description offers multiple realizations of a particular feature at a higher level, which is characteristic of *supervenience*.

(4) The description of features of a system at a particular level of description offers *neither necessary nor sufficient* conditions to derive the description of features at a higher level. This represents a form of *radical emergence* insofar as there are no relevant conditions connecting the two levels whatsoever.

For obvious reasons, class (4) is unattractive if one is interested in explanatory relations between different levels of description. Non-reductive property dualism (e.g., Davidson 1980) would be an example of radical emergence. By contrast, class (1) is extremely appealing if one is interested in simple explanations. The "received views" of reduction – as Batterman (2002) refers to them – fall into this class (e.g., Nagel 1961, Schaffner 1976).

From a contemporary point of view, classes (2) and (3) are viable alternative schemes for analyzing relationships between different levels of description. Supervenience relations, generally belonging to class (3),[1] have been extensively discussed on the basis of Kim's proposals (Kim 1993). Interestingly, Kim himself has recently argued that supervenience may be inadequate for capturing relations in the sciences (Kim 1998, 1999). This development has led to an emphasis on realization relations (e.g., Kim 1998, 1999, Crook and Gillett 2001, Gillett 2002).

In the remainder of this contribution we will focus our discussion on class (2), contextual emergence, which is less rigid than the strong form of reduction (1) on the one hand and provides more structure for interlevel relations than radical emergence (4) on the other.

3. Contextual Topologies and Asymptotic Expansions

A precondition for achieving a formal relation between descriptions at different levels is a well-defined concept of states and properties of the system considered at those levels. The algebraic approach in physics offers such well-defined concepts. For example, in algebraic quantum theory, properties are introduced as so-called observables forming a C^*-algebra[2] $\mathcal{A}$

[1]Some versions of supervenience require that changes in lower-level descriptions are both necessary and sufficient to bring about changes in a higher-level description. Such versions are indistinguishable from reduction (Kim 1998) and fall into class (1).

[2]A *-algebra is an algebra admitting an involution $^* : \mathcal{A} \to \mathcal{A}$ with the usual properties. A *-algebra is normed, if there is a mapping $||.|| : \mathcal{A} \to \mathbf{R}_+$ with the usual properties. A complete normed *-algebra is a

over the complex numbers which is not commutative in general. The associated concept of a state is introduced in terms of a positive normalized linear functional on $\mathcal{A}$. The state space of a fundamental theory in physics is chosen such that only the most basic assumptions are required for its definition. In other words, the state space is chosen as context-independent as possible.

Contexts are contingent conditions referring to the degree of "abstraction" at which a theoretical framework is formulated. Each description requires "abstracting from", i.e. disregarding, those details of a given system (and its environment) which are to be considered irrelevant. Needless to say, declaring particular features as irrelevant is not a universally prescribed procedure, but must be tailored to particular purposes or interests. Features which are irrelevant in a particular context may be highly relevant in another.

For instance, temperature is an example of a feature that is relevant in thermodynamics but irrelevant in Newtonian or statistical mechanics. Light rays are relevant in geometric optics, but they are irrelevant in Maxwell's electrodynamics. The chirality of molecules is relevant in physical chemistry, but it is irrelevant in a Schrödinger-type quantum mechanical description. Nevertheless, there are strategies for implementing the contexts due to which temperature is relevant in thermodynamics, due to which rays are relevant in geometric optics, and due to which chirality is relevant in physical chemistry, at the level of statistical mechanics, of electrodynamics, and of quantum mechanics.

A natural way to represent contexts of these kinds is the modification of the original topology of the lower-level state space to a *contextual topology* (Primas 1998). The finest topology corresponds to the most fundamental context,[3] e.g. given by "first principles," while coarser topologies represent an increasing amount of contextual information not encoded in first principles. The key idea of relating properties at different levels of description to each other is to specify the difference between the descriptions in terms of the contextual topologies of their corresponding state spaces.

Implementing a particular set of contexts as a contextual topology is usually nontrivial. A powerful tool often used for this purpose are asymptotic expansions (see Friedrichs 1955, Dingle 1973, Berry 1994, Batterman 2002). In order to formulate such an expansion, a reference state, which represents essential features of the context, has to be specified in the lower-level state space of the fundamental description. If the expansion is singular (i.e., diverges) in the intrinsic, fine topology of the fundamental description as an appropriate parameter tends to some limit, this indicates the need for a change of topology. Examples for such parameters are the number of degrees of freedom for thermodynamics (thermodynamic limit), the wavelength for geometric optics (short-wavelength limit), or the electron mass divided by nuclear mass for physical chemistry (Born-Oppenheimer limit).

The crucial point then is to identify a new topology which regularizes the expansion such that it converges. This leads to a contextual topology of the state space which is coarser than the original finer topology of the fundamental theory. This contextual topology is contingent in the sense that it is not given by the original finer topology or any other elements of the fundamental theory. The closure of such new descriptions in the contextual

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Banach *-algebra. A C^*-algebra is a Banach *-algebra $\mathcal{A}$ with the additional property $||x^*x|| = ||x||^2$ for all $x \in \mathcal{A}$ (see Takesaki 2002, chap. I.1).

[3]For instance, in conventional quantum theory states are standardly represented in a Hilbert space endowed with the norm topology. This topology derives from the C^*-property as expressed in footnote 2.

topology generates new context-dependent features not defined in the original state space under the finer topology (see next section for an illustrative example).

Invoking a new contextual topology, then, accommodates novel features within a higher-level description rather than approximating them as a limiting case in the topology of a lower-level description. It is important to realize that "the task of higher level descriptions is not to approximate the fundamental theory but to represent new patterns" (Primas 1998, p. 87). In general, these patterns are not reducible to a more fundamental level in the strict sense of class (1). Such reducibility would mean that only the first principles of the fundamental description are needed to describe new patterns exhaustively. If higher-level contexts in addition to first principles must be considered in order to rigorously derive descriptions of these new patterns, reduction according to class (1) fails. In such cases, contexts are at least as important as first principles.

The procedure described so far provides a formal approach for a contextual emergence of higher-level features on the basis of contexts in addition to the terms of the lower-level description. Qualitatively new, emergent features, unavailable in the lower-level description, manifest themselves at the higher level. In somewhat different terms, the new state space with coarser topology can be considered to be partitioned (i.e. coarse-grained) in a way allowing the definition of new states, together with associated new observables, represented by the cells of its partition (cf. beim Graben 2004, Atmanspacher and beim Graben, this issue). In this terminology, the choice of a proper partition is not prescribed at the lower-level description, but depends on the purpose of the partitioning and is usually based on concepts that are foreign to the lower-level description. This is, for example, the core idea in the construction of a symbolic dynamics within the theory of dynamical systems. (See Lind and Marcus (1995) for a comprehensive introduction to the field of symbolic dynamics.)

In this sense we suggest considering emergent features within class (2) of the proposed classification scheme. The contextual emergence of such features can be physically motivated and made mathematically rigorous via contextual topologies. While necessary conditions for emergent features exist at the lower level, sufficient conditions, represented by contexts, do not exist at the lower level. Hence, emergent features cannot be derived or predicted from the lower level alone, even if exhaustive information concerning this level is assumed. Only the lower-level description *plus* the appropriate contextual topology renders emergent features (e.g., emergent properties) derivable or predictable.

4. Thermodynamic Equilibrium and Temperature

This section discusses an example of contextual emergence in enough detail to see how contexts can be introduced leading to contextual topologies and emergent properties. Moreover, it is shown how necessary conditions for the emergence of novel properties are related to lower-level descriptions, whereas contingent contexts, not available at the lower-level description, represent sufficient conditions leading to well-defined properties at higher-order levels of description.

Our example is the often discussed reduction or emergence, respectively, of thermodynamic properties such as temperature to or from properties at lower-level descriptions. The lower-level descriptions in this context are statistical mechanics and point mechanics. How are these levels of description related to thermodynamics?

First, the less controversial issue: The step from point mechanics to statistical mechanics is essentially based on the limit of (infinitely) many degrees of freedom. That is, particular properties of a system are defined in terms of a statistical description (e.g., for many particles) and make no sense in an individual description (e.g., for single particles). An example is the mean kinetic energy of a system of N particles, which can be calculated from the distribution of the momenta of all particles. Its expectation value is defined in the limit of infinitely many particles, assuming the applicability of limit theorems (e.g., the law of large numbers). Any thermodynamic property whose definition is based on a statistical description presupposes (infinitely) many degrees of freedom. This applies to several properties, and temperature is a paradigmatic example. The concept of temperature is meaningless for systems whose number of particles is too small.

The more controversial issue in discussing the reduction or emergence of temperature refers to the step from statistical mechanics to thermodynamics (cf. Compagner 1989). In many philosophical discussions it is argued that the thermodynamic temperature of a gas *is* the mean kinetic energy of the molecules which by hypothesis constitute the gas. According to Nagel, this leads to a straightforward reduction of thermodynamic temperature to statistical mechanics (Nagel 1961, pp. 341-345).

Such a rough picture, however, is a gross mischaracterization, based on a too generous treatment of important details. First of all, thermodynamic descriptions presume thermodynamic, or briefly thermal, equilibrium as a crucial assumption which is neither formally nor conceptually available at the level of statistical mechanics. Second, the very concept of temperature is fundamentally foreign to statistical mechanics and has to be introduced, e.g., on the basis of phenomenological arguments.

Thermal equilibrium is formulated by the zeroth law of thermodynamics: if two systems are both in thermal equilibrium with a third system, then they are said to be in thermal equilibrium with each other. Based on this equivalence relation, the phenomenological concept of temperature can be introduced in the usual textbook way. Since thermal equilibrium is not defined at the level of a mechanical description, temperature is not a mechanical property but, rather, emerges as a novel property at the level of thermodynamics.

In this sense, the concept of thermal equilibrium serves as a context providing conditions for a proper discussion of temperature. This context is available at the higher-level description of thermodynamics. It can be recast in terms of a class of distinguished thermal states, the so-called Kubo-Martin-Schwinger (KMS) states, at the lower-level statistical description. These states are defined by the KMS condition which is equivalent to a variational principle, representing the stability of a KMS state against local perturbations. [4] Hence, the KMS condition implements a higher-level context in terms of a lower-level stability condition distinguishing states that are thermal in the sense of the zeroth law of thermodynamics. The second law of thermodynamics expresses this stability criterion in terms of a maximization of entropy. (Equivalently, the free energy of the system is minimal in thermal equilibrium.) If a system is in a KMS state, then this state is the canonical Gibbs state,

[4]For more details concerning the significance of the KMS condition see Sewell (2002, chap. 5). The stability requirement imposed by the KMS condition is discussed in detail in Atmanspacher and beim Graben (this issue).

uniquely defining a parameter interpreted as a (inverse) temperature.

In the framework of an algebraic statistical mechanics description, KMS states serve as reference states for a Gel'fand-Naimark-Segal (GNS) construction. Such reference states are functionals on a fundamental, lower-level, algebra of observables. The GNS-construction gives rise to another, higher-level algebra of observables including thermodynamic temperature as a novel property of the system. Takesaki (1970) has shown that temperature emerges as a classical observable from an underlying quantum statistical description.

Temperature is then an element of an algebra $\mathcal{M}$ of contextual observables, where the context is introduced by the KMS state as a reference state plus the contextual topology induced by this reference state. Since mechanical descriptions are given by type I W^*-algebras and the contextual W^*-algebra $\mathcal{M}$ is of type III,[5] temperature cannot be an element of a mechanical description (Primas 1998). Hence, temperature is not reducible to statistical mechanics in any straightforward sense. Thermodynamic temperature is an example of a contextually emergent property, which is neither contained in nor predicted by the exhaustive lower-level mechanical description alone. However, given the lower-level mechanical description and an appropriate contextual topology based on the KMS state, thermodynamic quantities can be rigorously derived. The contextual topology is a contingent condition not implied by the lower-level topology as is not the concept of thermal equilibrium applicable at the lower level. This fits precisely the conceptual scheme of contextual emergence, where the emergent property is the temperature (or other thermal features) of thermodynamics.

5. Stability Principles for Contextual Emergence

After the detailed discussion of thermodynamic properties as exemplars for contextual emergence, it is worthwhile to step back and look at its general principles. Repeating the characterization of contextual emergence as given in section 2, the description of features of a system at a particular level of description offers *necessary but not sufficient* conditions to derive features at a higher level of description. In logical terms, the necessity of conditions at the lower level of description means that higher-level features *imply* those of the lower level of description. The converse – that lower-level features also *imply* the features at the higher level of description – does not hold in contextual emergence. This is the meaning of the absence of sufficient conditions at the lower level of description. Contingent contexts for the transition from the lower to the higher level of description are required in order to provide such sufficient conditions.

In the example of temperature, the notion of thermal equilibrium represents such a context. Thermal equilibrium is not available at the level of description of Newtonian or statistical mechanics. Using the KMS condition and the limit $N \to \infty$, temperature can be obtained as an emergent property at the level of a thermodynamical description. It is of paramount

[5] A W^*-algebra is a *-algebra which is isomorphic to a closed algebra of observables on a Hilbert space. A C^*-algebra $\mathcal{M}$ is a W^*-algebra if and only if it is the dual of a Banach space $\mathcal{M}_*$, where $\mathcal{M}_*$ is the predual of $\mathcal{M}$ (see Takesaki 2002, Chap. III.3). W^*-algebras can be classified by their central decompositions, i.e. by factors. A factor is of type I if it contains an atom. It is of type III if it does not contain any nonzero finite projection. It is of type II if it is atom-free and contains some nonzero finite projection. For more details see Takesaki 2002, p. 296).

importance for this procedure that the KMS state satisfies stability criteria [6] that are induced by the contextual condition of thermal equilibrium at the level of thermodynamics and can be implemented at the level of statistical mechanics.

Since the Newtonian and statistical mechanical levels of description are necessary to derive the higher-level property of temperature, principles or laws at these levels of description cannot be violated by any higher-level description incorporating temperature. That the Newtonian and statistical mechanical levels of description alone are not sufficient is recognized by the fact that they do not give rise to an algebra of observables including temperature unless additional contingent contexts are specified.

The significance of contextual emergence as opposed to reduction in this example is clear. It would be interesting to extend the general construction scheme for emergent properties to other cases. More physical examples are indicated and discussed, for example, in Primas (1998) and Batterman (2002). But the concept of stability might be useful as a key principle for the construction of a contextual topology and an associated algebra of contextual observables in examples even beyond physics.

One possible, and ambitious, case refers to emergent features in the framework of contemporary neuroscience. A particularly active field of research here is concerned with the emergence of new features at the level of neuronal assemblies from lower-level features of individual neurons. Particular interest in this issue derives from the fact that cognitive capabilities are usually correlated with the activity of neuronal assemblies, but detailed neurobiological knowledge refers mainly to the properties of individual neurons. Closing the gap in our understanding of the relation between neuronal assemblies and individual neurons could contribute significantly to understanding neurobiological correlates of consciousness.

As a possible framework for research in this area, the scheme of contextual emergence might be fruitfully applied as follows. Novel features at the (higher) level of neuronal assemblies would have necessary but not sufficient conditions at the (lower) level of neurons. In order to identify contexts providing such sufficient conditions, those among the many possible assembly features which are relevant or interesting as emergent features must first be identified. Assuming that stability criteria play a role analogous to physical examples, techniques of modeling assemblies in terms of generalized potentials with particular stability properties and corresponding relaxation times or escape times suggest themselves. This can be implemented easily for powerful modeling tools such as neural networks (Anderson and Rosenfeld 1989) or coupled map lattices (Kaneko and Tsuda 2000).

Contextual emergence might even be a viable scheme to address relations between the neurobiology of the brain at various levels on the one hand and cognitive or psychological features – in other words: to address the relation between material (brain) and mental (consciousness) features. In another paper (Atmanspacher and beim Graben, this issue) concrete applications in cognitive neuroscience are elaborated in detail.

[6] The notion of the stability of a system as used here covers both structural and dynamical aspects, reflected by the invariant ergodic measure of the dynamics of the system and the existence of attractors, respectively. A more detailed account, which is beyond the scope of this contribution, can be found in Atmanspacher and beim Graben (this issue).

6. Summary

The goal of reduction is to derive the description of higher-level features of a system exhaustively in terms of the description of features at the most fundamental level of physical theory, no matter how remote the higher level is from that most fundamental level. The implicit assumption in this program is that the description of all features which are not included at the fundamental level can be constructed or derived from this level without additional input. However, many physical examples pose serious difficulties for this program. For instance, temperature is a novel property emerging from a more fundamental statistical mechanical description, but it is not derivable from this description alone.

The concept of contextual emergence addresses such situations properly. Contextual emergence is characterized by the fact that the lower-level description provides necessary, but not sufficient conditions for higher-level descriptions. The presence of necessary conditions indicates that the lower-level description provides a basis for higher-level descriptions, while the absence of sufficient conditions means that higher-level features are neither logical consequences of the lower-level description nor can they be rigorously derived from the lower-level description alone.

Hence, the notion of reduction is inapplicable in these cases. Sufficient conditions for a rigorous derivation of higher-level features can be introduced through specifying contexts reflecting the particular kinds of contingency in a given situation. Expressing these contexts in the lower-level description induces a change of the topology of the associated state space. There is a mathematically well-defined procedure for deriving higher-level features given the lower-level description plus the contingent contextual conditions.

A key ingredient of this procedure is the definition of some type of stability condition (e.g., the KMS condition) based on considerations required to establish the framework of a higher-level description (e.g., thermal equilibrium). This condition is often implemented as a reference state with respect to which an asymptotic expansion is singular in the lower-level state space. Regularizing the expansion provides a novel, contextual topology in which novel, emergent features can be rigorously introduced. In the thermodynamic example, this procedure is represented by the GNS-construction.

We propose that contextual emergence and the associated identification of appropriate stability conditions may have applications in other domains such as biology and psychology, and, ultimately, in the relationship between the physical and the mental. Concrete ways of how this can be achieved have been worked out elsehwere (Atmanspacher and beim Graben, this issue).

Acknowledgments

We appreciate the helpful suggestions of two referees, due to which an earlier version of this paper has been improved.

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