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# Broad Views of the Philosophy of Nature – Riemann, Herbart, and the “Matter of the Mind”

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*This paper deals with an attempt of the mathematician Bernhard Riemann to develop an outstandingly broad view of the philosophy of nature encompassing basic phenomena of both the material and the mental world. Riemann’s draft is traced in its main aspects, and is accompanied by a comparison with certain chapters in the philosophical writings of Herbart that were particularly relevant to Riemann’s conception of mathematics and science on the whole. This applies, in particular, to the epistemological background and to Herbart’s theory of the Self.*

## 1. Introduction

The mystery of the relationship between mind and matter, the material and the mental or spiritual world,<sup>1</sup> has attracted the interest of philosophers through the ages. The related discussion customarily proceeds in general terms. Although concepts and facts contributed by the diverse subject areas have increasingly been taken into account in the past decades, serious attempts to actually develop a comprehensive formal framework for the mind-matter problem seem to be very rare. (However, see Primas, 2007.) Our focus here is on a not widely recognized such attempt undertaken by the well-known mathematician Bernhard Riemann (1826-1866). The following citation (translations to English are the author’s if not stated otherwise) immediately leads in medias res. It is quoted from the section “*Neue mathematische Principien der Naturphilosophie*” contained in the (posthumously published) appendix “*Fragmente philosophischen Inhalts*” to Riemann’s Collected Mathematical Works (1892) [RCMW]. I call it:

*Riemann's Other Hypothesis* [OH]:<sup>2</sup>

“[...] I hypothesize that space throughout is filled with a stuff that continually flows into the ponderable atoms and disappears there from the phenomenal world (material world). Both hypotheses may be substituted by the single one that in all ponderable atoms, stuff from the material world continually enters into the spiritual world. The cause for the disappearance of the stuff is to be looked for in the spiritual substance built there immediately before, and the ponderable bodies hence are the location where the spiritual world takes action in the material world.”

“[...] mache ich die Hypothese, dass der Weltraum mit einem Stoff erfüllt ist, welcher fortwährend in die ponderablen Atome strömt und dort aus der Erscheinungswelt (Körperwelt) verschwindet. Beide Hypothesen lassen sich durch die Eine ersetzen, dass in allen ponderablen Atomen beständig Stoff aus der Körperwelt in die Geisteswelt eintritt. Die Ursache, weshalb der Stoff dort verschwindet, ist zu suchen in der unmittelbar vorher dort gebildeten Geistessubstanz, und die ponderablen Körper sind hiernach der Ort, wo die Geisteswelt in die Körperwelt eingreift.”

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<sup>1</sup> Our nomenclature adheres to the common (Cartesian) terms of the debate. For an incisive analysis of the mind/matter duality based on a radically different approach see Wackermann (2008).

<sup>2</sup> ‘Riemann’s Other Hypothesis’ [OH] is an allusion to a mathematical conjecture known as *the* Riemann Hypothesis [RH]. The RH counts among the greatest open problems in mathematics and generated, in stark contrast to the highly speculative OH, a great deal of research. Here we use ‘OH’ simply as a shortcut crediting the importance Riemann himself attached to the subject. Heaving the OH to the level of the RH is not intended in any way.

Riemann states his OH in an audacious attempt at unifying not only the basic physical phenomena known at his time, notably gravitation and light; such had also been attempted by Euler, for instance. Riemann goes on to encompass at the same time mental phenomena such as the appearance, vanishing, or recollection of thoughts, perceptions, etc. And he does so guided by, and pursuing, general conceptual considerations that themselves flow from overarching epistemological ideas. Those ideas were strongly influenced by the writings of the philosopher, psychologist, and educationist Johann Friedrich Herbart (1776-1841), the first, too, to envisage even the possibility of a discipline ‘mathematical psychology’; see Boudewijnse et al. (1999, 2001) and Murray & Bandemir (2002). Riemann also picked up from Herbart the concept of ‘representational series’ (*Vorstellungsreihen*) which in his speculations reappear in a slightly different way as ‘spiritual substance’ or ‘spiritual masses’ (*Geistessubstanz, Geistesmassen, Vorstellungsmassen*).

Riemann’s presentation in the *Fragmente philosophischen Inhalts* (hereafter: *Fragmente*) is very sketchy, the parts of interest to us covering about 20 pages. Nevertheless, it touches a wide range of subjects ranging from epistemology, physics, psychology, to physiology. The goal of this paper is to present these matters in a way so as to make the OH better comprehensible; and to contrast Riemann’s outline with corresponding places taken from the writings of Herbart in order to highlight the parallels, and hence, the *extent* to which Riemann was influenced by Herbart.

This influence has been known for a long time, not the least because Riemann himself mentioned it. Its nature was not so clear, however, and initially was suspected to mainly cover Riemann’s philosophy of space. The picture changed when E. Scholz could show, by analysing Riemann’s notices of his studies of Herbart available in the *Riemann Archiv* at Göttingen, that “Herbart influenced Riemann much more in his epistemology and the comprehension of science” (Scholz, 1982).<sup>3</sup> There are now several studies confirming this view; see, e.g., Scholz (1992), Smadja (2004), Banks (2005). Much of the present paper’s contents is covered more or less, in greater generality or detail, by these works, furthermore by Laugwitz (1996) and the Herbart studies Boudewijnse, Murray & Bandomir (1999, 2001), Murray & Bandomir (2002). What may be different here is, besides the arrangement of the material around Riemann’s *Fragmente* and the emphasis on the OH: the presentation of the ‘layers’ science is dealing with; a delineation of Herbart’s theory of the Self; and its (proposed) connection to Riemann’s concept of the ‘Agents’.

Let us yet add some notes on the question what Riemann may have attracted to the enterprise that led him to the statement of the OH. First, Riemann’s thinking was firmly grounded in epistemology, and he is in fact regarded as being responsible for a fundamental change in the view of mathematics, from the algorithmic, calculatory, quantitative and special to the conceptual, structural, qualitative and general. This topic is expounded in depth in Laugwitz (1996). Second, Riemann maintained a strong interest in the philosophy of nature including physiological, psychological, also religious matters throughout his lifetime. (Cf. the leader introducing the *Fragmente* and Riemann’s vita; RCMW, pp. 507-508, 539 et seq.) Third, as pointed out by, e.g., Laugwitz (1996), Murray & Bandemir (2002), and Smadja (2004), Riemann worked in a period of rapid scientific progress, where newly discovered phenomena and unexplained experimental results called for novel concepts and approaches, and for a revision of what might be conceived as the unity of nature. As for a few catchwords and names: Oersted,

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<sup>3</sup> While correcting previously held views, Scholz’s findings re-enthroned Riemann’s own stance in this matter. That stance is clearly stated in the following citation (RCMW, p. 508; Scholz’s translation): “The author is a Herbartian in psychology and epistemology (methodology and eidology); in most cases, however, he cannot agree with Herbart’s natural philosophy and the metaphysical disciplines (ontology and synechology) referring to it.” (‘Eidology’ refers to Herbart’s theory of the Self, ‘synechology’ to the philosophical foundation of the concept of space.)

Faraday and electricity/magnetism, Weber, Fechner and psychophysics, Helmholtz, du Bois-Reymond and physiology. A particular case in point could be Helmholtz's lecture "*Über die Erhaltung der Kraft*" delivered 1847 which may have conveyed on Riemann the idea of possible transformations between the diverse natural forces. Not to speak, finally, of the open intellectual atmosphere of the time, with its vivid debates and polemics among philosophers and scientists, including also an interested public.

## 2. Epistemology

### 2.1 Herbart

Herbart, in the introduction to his two volume work with the descriptive title "Psychology as a Science, Newly Founded on Experience, Metaphysics, and Mathematics," sets off with a highly condensed statement of its purpose:

"The goal of this work is to bring about a science of the soul that resembles the science of nature; inasmuch as this research throughout presupposes a lawful connection in the phenomena and searches after it by sifting the evidence, by cautious conclusions, by venturing, checking, rectifying hypotheses, finally, whenever feasible, by pondering magnitudes and by calculation."

*"Die Absicht dieses Werkes geht dahin, eine Seelenforschung herbeizuführen, welche der Naturforschung gleiche; in so fern dieselbe den völlig regelmässigen Zusammenhang der Erscheinungen überall voraussetzt, und ihm nachspürt durch Sichtung der Thatsachen, durch behutsame Schlüsse, durch gewagte, geprüfte, berichtigte Hypothesen, endlich, wo es irgend sein kann, durch Erwägung der Grössen und durch Rechnung."*

The first part reflects Herbart's desire to open up for psychology the prospect of advancements similar to those that had been made by the natural sciences up to his time. He continues by indicating that this could be achieved by borrowing the recipe for success of the natural sciences: to start out from the phenomena on one, and from basic principles on the other hand, and to relate both to each other using a clear-cut methodological strategy. In current language the "basic principles" could be regarded as a system of axioms: taken together they should be parsimonious, have an inherent certainty on their own ("*soll eigene Gewissheit ursprünglich haben*"), and "give rise to other certainty" by the application of suitable methods.

What are the principles of psychology? According to Herbart, they are "those facts of the mind from which the laws of what is happening within us can be discerned." (More on this in Section 4.2.) Our knowledge of "the facts of the mind" inevitably is deficient and needs to be backed up by pure reasoning, or by speculation if necessary. The speculation has to infer from the "immediately Given" to the underlying "Real," departing from the "insight that the Given could not show up if there was no Real." Further, "What science knows better than experience, it only can know by the circumstance that the experienced is unintelligible without assuming the hidden." Science depends on experience, for it is there where it "must come across and discern the traces of what budes and acts behind the curtain." However, it also has to go beyond pure empiricism by developing concepts for "the hidden," in which latter the explanation is to be sought for the phenomena accessible to experience. Importantly, one should, and can, trust in the conclusions obtained by reasoning from principles that complement what we know by experience; fluctuant and uncertain are only the phenomena. Moreover, Herbart would not accept leaps or gaps in arguing, each point should follow each other quasi continuously.

Here is a tentative summary of Herbart's conception of 'psychology as a science':

- determine the relations observable within the phenomena;
- complement these by introducing (and reworking if necessary) concepts for the unobservable;
- using appropriate, logically consistent methods.

## 2.2 Riemann, Hertz, and ‘Pictures of the World’

Similarly to Herbart’s introduction, the section in the *Fragmente* entitled “epistemology” (*Erkenntnistheoretisches*) starts with a programmatic statement.

“Science [of nature] is the attempt to understand nature through precise concepts.”

„Naturwissenschaft ist der Versuch, die Natur durch genaue Begriffe aufzufassen“

Science is characterized as a *process* involving two essential components, the perceptions or observations (*Wahrnehmungen*), and the concepts (in fact: a framework of concepts) serving to describe nature. Science becomes a process inasmuch as concepts have to be modified or adjusted whenever there appear phenomena that would have to be considered impossible or improbable on the current conceptual basis. The reworking process ends successfully, for the moment being, only if phenomena ultimately appear necessary or probable.

Where do the appropriate concepts come from? Riemann emphasizes that they are arrived at through such steadily continuing reworking processes of earlier frameworks. In particular, one does not have to invoke a specific constitution of human mind providing the appropriate concepts a priori—a remark directed expressly against Kant. It is palpable here (as elsewhere—the connections usually are evident enough to not require explicit mentioning) that Riemann’s epistemological notes follow Herbart very closely, who more or less *defined* philosophy as a continuous process wherein systems of concepts are being reworked so as to become more and more adequate for the subject under study (*“Methode der Beziehungen”*). Riemann goes on with an equally programmatic pair of questions and answers.

Q1: When is our conception of the world true?—A1: If the coherence of our concepts corresponds to the coherence among the things.

Q2: Wherefrom is the coherence among the things to be found?—A2: From the coherence of the phenomena.

This implies, first, that Riemann discriminates between the ‘real’, the ‘things themselves’, and their phenomena—how they appear to us. Moreover, it implies that concepts should be built both at the level of the phenomena and the things. This is implicit at least in the following citations further developing Riemann’s views. “The elements of the picture of the world differ essentially from the corresponding elements of the world themselves. The former lie within, the latter outside ourselves. However, the interrelations between the elements of the picture and the depicted have to coincide if the picture is to be a true one.”

*“Die Elemente des Bildes von der Welt sind von den entsprechenden Elementen des abgebildeten Realen gänzlich verschieden. Sie sind etwas in uns; die Elemente des Realen etwas ausser uns. Aber die Verbindungen zwischen den Elementen im Bilde und im Abgebildeten müssen übereinstimmen, wenn das Bild wahr sein soll.”*

In short: the *structural relations* between the concepts at the levels of the phenomena and of the things should match, whereas nothing is (to be) said about the nature of the ‘things themselves’. According to Q2/A2, the structural relations between the things have to be uncovered from the “coherence of the phenomena,” or, in fact, by pondering the “observed coherence of the relations between magnitudes.” Thus ultimately, the *quantitative, functional* relations prevailing in the phenomena are to provide the cue to the sought-after relations between the elements of the real. Note that a purely empiricist approach would lack the “picture” and deal only with the “elements of the world.”

The following quotation highlights once more Riemann’s idea of the gradual, continuous advancement of science. Almost ideal instances thereof were delivered (later on) by the embedding of classical mechanics into relativistic and quantum mechanics, respectively.

“The truth of the picture is independent of its degree of acuteness. [...] If one element of the picture is replaced by a group of more acute elements, such that its properties partially follow from their interrelations and can thus be understood partially, then while our insight regarding the coherence of things is enhanced, the earlier conception need not necessarily be declared erroneous.”

*“Wird ein Element des Bildes durch eine Gruppe von feineren Elementen ersetzt, sodass seine Eigenschaften theils aus ihrer Verbindung sich ergeben und also zum Theil begreiflich werden, so wächst dadurch zwar unsere Einsicht in den Zusammenhang der Dinge, aber ohne dass die frühere Auffassung für falsch erklärt werden müsste.”*

Riemann’s statements bear a striking resemblance with an equally matter-of-principle passage by Heinrich Hertz (1857-1894) appearing in the introduction of his “Principles of Mechanics” (Hertz, 1894):

“We make for ourselves inner pictures or symbols of the external things, in such a manner that the logically necessary sequels of the pictures always are the pictures of the, by nature necessary sequels of the depicted things. [...] The pictures<sup>4</sup> we are speaking of are our representations of the things; they share with the things the essential compliance consisting in the above requirement, however, any further agreement beyond that requirement is unnecessary. In fact, we do not know, and have no means to learn, whether our representations of the things coincide with the latter in anything else than just that fundamental relationship.”

*“Wir machen uns innere Scheinbilder oder Symbole der äusseren Gegenstände, und zwar machen wir sie von solcher Art, dass die denotwendigen Folgen der Bilder stets wieder die Bilder seien von den naturnotwendigen Folgen der abgebildeten Gegenstände. [...] Die Bilder, von denen wir reden, sind unsere Vorstellungen von den Dingen; sie haben mit den Dingen die eine wesentliche Übereinstimmung, welche in der genannten Forderung liegt, aber es ist für ihren Zweck nicht nötig, dass sie irgendeine weitere Übereinstimmung mit den Dingen haben. In der Tat wissen wir nicht, und haben auch keine Mittel zu erfahren, ob unsere Vorstellungen von den Dingen mit jenen in irgend etwas anderem übereinstimmen, als allein in eben jener einen fundamentalen Beziehung.”*

Note that Herbart, Riemann, as well as Hertz dismiss as infeasible any attempt at getting at the bottom of the things, and emphasize structural relations instead.

It is tempting to try elementary concepts of mathematical category theory (e.g., Adámek et al., 1990) on the epistemological reflections outlined above. Riemann’s (and Hertz’s) notes prompt the idea of a functor that connects two categories: the category P of the observable phenomena, and the category M of our conception (*viz.* ‘model’) of the ‘underlying real’. Roughly, a category consists of ‘objects’ and of ‘arrows’ (fulfilling certain composition rules), where the latter describe the relations between the objects. A (covariant) ‘functor’ is a mapping between two categories that maps both objects and arrows of a category C1 to the objects and arrows of another category C2 such that the fundamental relations coded in the arrow structure of C1 are preserved in its functorial image within C2. The objects of the categories P and M would then correspond to the respective concepts, the arrows to the relations-between-concepts. Riemann’s and Hertz’s basic demand what the ‘picture’ should accomplish could then be formalized as follows: the systems of the concepts (objects) and relations-between-concepts

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<sup>4</sup> The meaning of ‘picture’ is different in Riemann and Hertz: Hertz’s ‘pictures’ (as they are understood here) correspond to Riemann’s ‘elements of the picture’ or ‘concepts’, whereas Riemann’s ‘picture’ comprises the whole scenario, as it were.

(arrows) making up the categories P and M should be consistent in the sense that they admit a functor from M to P.

The ‘direction’ of the functor, from M to P, accounts for the requirement that the conceptualization at the level of the underlying real should be such that the trace it leaves within P should be consistent with the relations (arrows) prevailing there. On the other hand, the concepts and relations-between-concepts of the category P serve to *describe* the phenomenal world, hence should be consistent with experience. This is the impetus driving the oscillations of the scientific process: the category M which serves to *explain* the observable phenomena has to be adjusted so as to reflect the structure of the category P that in turn has to be compared with experience—which, again, may suggest necessary adjustments within M and P. The process is conceived as proceeding gradually in a stepwise manner. In particular, it is not assumed to terminate. Interestingly also, none of the three authors requires the ‘picture’ to be complete. It is not demanded that the phenomenal description via P captures all phenomena, nor even that the conceptual explanation via M accounts for the full structure of P (i.e., the functor need not be an isomorphism)—compare Riemann’s statement “The truth of the picture is independent of its degree of acuteness.”

One will have noticed that the above presentation slightly differs from the one of Herbart, Riemann, and Hertz. While in the latter there are two ‘layers’ corresponding to what is “before / behind the curtain,” above there are actually three layers: while the “behind the curtain” is referred to by category M, the “before the curtain” is split into its conceptualization (category P) and into ‘(comparison with) experience’. The idea here is that the phenomenal world “before the curtain” may not be ‘immediately given’ such as, e.g., a table or the moon, but also involves objects such as ‘electron’, ‘energy’, or, in psychology, ‘introversion’, etc.<sup>5</sup> The latter themselves need conceptualization or ‘operationalization’ if they are to become useful objects of the category P. ‘Comparison with experience’ then means that the relations assumed (predicted, stipulated, etc) to hold between the objects (i.e., the arrows of P) are checked against the (as a rule: approximative only) relations found in empirical data.

The categories M and P have been introduced to elaborate on the texts of Herbart, Riemann, and Hertz.<sup>6</sup> One may ask here whether, or to what extent, M and P could be identified. Obviously, this is related to the question *where* “the curtain” is located.

Thus far Riemann’s considerations are concerned with static relationships, lacking the flavor of time and cause inherent in Hertz’s ‘sequels’. Such come into the focus in a subsequent, brief fragment entitled *Causalität*, where Riemann very abstractly deals with what he calls an ‘Agens’. The Agens is discussed there as the fundamental principle underlying the transitions—which are always conceived as continuous—of a ‘thing’ from one state to another. Assuming a kind of conservation or stability principle—“a thing would persist if nothing else would interfere”—Riemann proposes the Agens as supplying the cause for any change. The essence of the Agens determines its action, which is “striving to conserve or reconstruct itself.” Riemann concludes that an Agens cannot be a ‘thing’ but has to be a “state or proportion” (*Zustand oder Verhältniss*; RCMW, p.524).

Despite this qualification, the notion apparently has a wider scope encompassing active as well as passive aspects. Thus “state or proportion” implicitly refers to ‘something’ in that state or proportion. That ‘something’ represents—not itself but only its state—a passive part suffering

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<sup>5</sup> For an elaboration of this idea cf. Wackermann (2008, pp. 200-202).

<sup>6</sup> The sketch is not claimed to faithfully represent the views of Herbart, Riemann, or Hertz. In particular, Hertz’s understanding of the ‘pictures’ may have been rather different (Heidelberger, 1998).

perturbations from an active part, “actio”, whose action consists in “striving to conserve or reconstruct itself”—‘itself’ again understood as a (previous) state. As will become more clear later on, that action may be conceived of as springing from a kind of inner strain that results from the deviation of the actual from previous states. In this way the Agens as “state or proportion” *re-acts on itself*, thus encompasses both its active and passive aspects simultaneously. Still, distinguishing these two aspects of the Agens is helpful conceptually. A ‘something’ as the carrier of a state plays at most an auxiliary role in this description, hence is not needed for the intrinsic determination of the concept ‘Agens’. However, such carriers do play a role when it comes to concrete applications of the concept in science.

### 3. Physics

Physical topics are dominant in the section of the appendix of RCMW entitled "New mathematical principles of the philosophy of nature" (referred to as *Neue Principien* subsequently). This is a coarse draft of a paper apparently intended for publication by Riemann, but actually never published. The added footnote “Found on 1<sup>st</sup> March 1853” indicates that Riemann may have thought of it as an important discovery. It starts with laying down its program:

“[The] purpose [of the paper] is to penetrate into the inner of nature, beyond the foundations of astronomy and physics layed by Galilei and Newton.”

*"[Der] Zweck [des Aufsatzes] ist, jenseits der von Galiläi und Newton gelegten Grundlagen der Astronomie und Physik ins Innere der Natur zu dringen."*

Riemann thus aims at more than just a description, but at an explanation of nature, in fact from its inner structure. He wants to explain in this manner not only the phenomena of gravitation and light. His reflections prompt him to state the OH, so that mental phenomena and “spiritual substance” are included within the same framework. One should recall that at Riemann's time there were two fundamentally different ideas about space and the movement of the planets:—Newtonian mechanics, where the planets are considered as point masses moving in an empty, absolute space according to Newton's laws of motion;—the aether hypothesis going back to Descartes and Huygens, and favored by Euler, according to which space throughout is filled with an extremely subtle and elastic, fluid-like substance called ‘aether’.

The aether is assumed to be susceptible to the slightest jars and able to transmit forces or actions locally. It thus helps to avoid an unpleasant apparent consequence of Newtonian mechanics, namely that the gravitational forces would act instantaneously at long distances, without any intermediate medium carrying them on.<sup>7</sup>

Riemann, too, wants to circumvent mysterious actions at a distance and postulates the existence of a space-filling stuff of which he assumes that it behaves like an incompressible homogeneous fluid without inertia. He largely avoids using the term aether and rather speaks of ‘stuff’ (*Stoff*), but he clearly adopts the aether hypothesis. Often he speaks of *Stofftheilchen* if he wishes to indicate that infinitesimally small portions of the aether are being considered. A key point now is his further assumption that this aether stuff has a dynamically changing inner state

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<sup>7</sup> The debate on this point is sophisticated. In fact, as Riemann was well aware of, Newton himself rejected the notion of action at a distance as a “great absurdity” (RCMW, p. 534). Instead, Newton invoked a “certain extremely subtle principle” (*spiritus subtilissimus*) apparently serving a similar purpose as the aether; cf. Newton (1988, pp. 230-231).

or form, and he then goes on asking what these form changes should be like if they were to produce the effects of gravitation and light.

Where do the form changes of those *Stofftheilchen* come from? The answer is: from their own history, as it were. According to Riemann the dynamics is due to a force stemming from the previous forms of the *Stofftheilchen*, precisely: from the respective *difference* to the present form. (The effect of that deviation is the smaller, he assumes hereby, the farther it lies back in time.) The difference then represents a kind of strain causing the form changes by virtue of a minimum action principle (RCMW, p. 527).

The mathematical treatment is only sketched. Riemann ends up with a decomposition of the form change into the change of a volume element and a line element. Gravitation and the propagation of light are traced back to the resistance of the *Stofftheilchen* against changes of the volume and line elements, respectively. The section subsequent to the *Neue Principien* entitled “Gravitation and light” furnishes slightly more detail, and ends with the statement of a variational principle from which the time-dependent potential and the velocities of the aether movement would yet have to be determined.

Three remarks: First, Riemann here basically deals with differential forms depending on three space and one time component. Therefore, it is quite plausible when A. Speiser (1927) speculates that these considerations were the starting point ultimately leading to Riemann's famous habilitation lecture *Über die Hypothesen welche der Geometrie zu Grunde liegen*. Not to speak of the vicinity in time: the *Neue Principien* are dated March 1853, the lecture was given in June 1854.

Secondly, a differential form attaches to each point geometrical properties that in fact require an infinitesimal neighbourhood for proper definition. If that is left out of consideration, it looks as if the point itself were capable of carrying structure, which naturally then would appear as its ‘inner structure’, hence could be seen as allowing a glimpse into “the inner of nature.” This view paraphrases the related interpretation of Smadja (2004), according to which the ‘ponderable atom’ may not correspond to a point, but rather to an infinitesimal parallelepiped as it is required for the construction of partial differential equations, the type of equations physical laws usually assume.

Third, Riemann's mathematical treatment of the form changes of the aether implements almost one-to-one the abstract treatment of the Agens in his notes on epistemology. In fact, in *Neue Principien* the active aspect of the Agens appears in the habit of a minimum action principle characterized by the property that it “[...] strives to keep the deviation of its state of motion (*Bewegungszustand*) at time  $t+dt$  from its state of motion at time  $t$  as small as possible;” or, briefly, by the “striving of its state of motion to conserve itself.” (These citations are from *Molecularmechnik*, a section which precedes, and prepares, the *Neue Principien*; RCMW, pp. 526-528.) The latter formulation dovetails perfectly with the characterization of the Agens—in fact, of its *action*—in the section *Causalität* by its “striving to conserve or reconstruct itself.” Importantly also, it is at the fundamental level of the Agens in its dual physical incarnation as an action principle and the aether (as the carrier of the states on which the principle acts)—i.e., “behind the curtain”—, where the explanation is sought for the laws governing the observable phenomena of gravitation and light. Below, the Agens will reappear in a different habit when we come to Herbart's concept of the Self.

To summarize so far: Riemann attempts to explain both gravitation and light propagation by local actions, via the form changes of some kind of aether. But what is the connection between the aether stuff and the gravitational forces? Riemann appears to suggest that the aether corresponds to a kind of potential, the gradient of which gives the direction of the aether movement—on one hand. The connection then comes from the postulate that the direction of the

gravitational force, too, should be proportional to that gradient (*Gravitation und Licht*; RCMW, p. 533.)

The situation invokes the idea of a flow of the fluid-like aether stuff towards the ‘ponderable masses’, at which point Riemann, according to a suggestion of Speiser (1927), may have faced a problem. Namely: what happens to the aether stuff that continually flows into a ponderable atom? Since there is only inflow and no outflow, there appears to remain no other resolution as to assume that the stuff just disappears at the atom. Hence the OH: “that space throughout is filled with a stuff that continually flows into the ponderable atoms and disappears there from the phenomenal world.” If one further sticks to the idea that what vanishes is really a kind of stuff how subtle it may be, then a conservation law in a very general sense might apply and suggest that the stuff continues to exist further somehow, though not in a material form. Hence Riemann's reformulation saying that “in all ponderable atoms, stuff from the material world continually enters into the spiritual world.”

Diverging from the hydrodynamic picture, Smadja (2004) suggests that the concept of a stuff whose “mode of motion” (*Bewegungsform*) carries gravitational forces and light propagation foreshadows the concept of a field as developed later on by Maxwell. Accordingly, one should not imagine the ponderable atom as a kind of sink devouring aether stuff, but rather conceive of the aether as transmitting a certain action quantity by means of the pressure it exerts near an atom. This notion appears consistent with the interpretation of the aether as a kind of potential (and of the atoms as singularities of the gravitational field intertwined with it).

## 4. Psychology

### 4.1 Riemann's ‘spiritual masses’

Thus far one well could consider the “spiritual world” or “spiritual substance” figuring in the OH as a somewhat eccentric way of expressing intuitions and matters of an essentially physical and mathematical origin. However, preceding the statement of the OH there is a passage appealing to psychological introspection.

“The origin for the general laws of motion for ponderabilities [...] lies in their inner condition. Let us try to deduce it from our own inner perception by analogy. There continually appear in us new representational masses that again vanish very quickly from our consciousness. We observe a persisting activity of our soul. Something permanent underlies each of their acts that pops up at particular occasions (via commemoration), without having a lasting influence on the phenomena. Thus at each moment something permanent enters into our soul (with each act of thinking) which, however, exerts no lasting influence on the world of the phenomena. Hence, there is something permanent underlying each act of our soul that enters our soul with this act, but at the same moment disappears completely from the world of phenomena.”

*“Der Grund für die allgemeinen Bewegungsgesetze für Ponderabilien [...] liegt im inneren Zustande derselben. Versuchen wir aus unserer eigenen Wahrnehmung nach der Analogie auf denselben zu schliessen. Es treten in uns fortwährend neue Vorstellungsmassen auf, welche sehr rasch wieder aus unserem Bewusstsein verschwinden. Wir beobachten eine stetige Thätigkeit unserer Seele. Jedem Act derselben liegt etwas Bleibendes zu Grunde, welches sich bei besonderen Anlässen (durch die Erinnerung) kundgibt, ohne einen dauernden Einfluss auf die Erscheinungen auszuüben. Es tritt also fortwährend (mit jedem Denkact) etwas Bleibendes in unsere Seele ein, welches aber auf die Erscheinungswelt keinen dauernden Einfluss ausübt. Jedem Act unserer Seele liegt also etwas Bleibendes zu Grunde, welches mit diesem Act in unsere Seele eintritt, aber in demselben Augenblick aus der Erscheinungswelt völlig verschwindet.”*

(In passing, the passage in the middle—the three sentences preceding the last one—is interesting. It shows that Riemann feels his way very cautiously on this ground. He repeats the respective preceding idea again and again, with only minute steps ahead from sentence to sentence, until he arrives at the decided last statement.)

Still, it seems that Riemann just tries to motivate his ultimately mathematical idea by an analogy—as he says, *nach der Analogie*. However, he goes ahead writing “Guided by this fact (*Thatsache*), I hypothesize [...],” and then comes his OH. So, Riemann's stance appears rather ambiguous at this point. There are hints, however, suggesting that he took the connection with psychological processes more serious than just as an analogy. Thus in *Neue Principien* Riemann wrote:

“There continually appear in us new representational masses (*Vorstellungsmassen*) that again vanish very quickly from our consciousness.”

Part I of the *Fragmente* starts as follows:

“With each simple act of thinking, something permanent, substantial enters our soul. To us, this substantial appears as a unity, however, (inasmuch it represents something extended in space and time) it seems to contain an inner manifold; I therefore call it ‘spiritual mass’.—Hence all thinking is generation of new spiritual masses. The spiritual masses entering our soul appear to us as representations; their varied inner condition implies their different quality.”

“*Mit jedem einfachen Denkact tritt etwas Bleibendes, Substantielles in unsere Seele ein. Dieses Substantielle erscheint uns zwar als eine Einheit, scheint aber (in sofern es der Ausdruck eines räumlich und zeitlich ausgedehnten ist) eine innere Mannigfaltigkeit zu enthalten; ich nenne es daher ‘Geistesmasse’.—Alles Denken ist hiernach Bildung neuer Geistesmassen. Die in die Seele eintretenden Geistesmassen erscheinen uns als Vorstellungen; ihr verschiedener innerer Zustand bedingt die verschiedene Qualität derselben.*”

And later on:

“The spiritual masses require no material carrier to endure, and they have no lasting impact onto the phenomenal world. Therefore, they have no relationship whatsoever to any part of matter and do not reside anywhere in space. However, all appearance, emergence, all forming of new spiritual masses and all coalescence of them does require a material carrier. Therefore, all thinking occurs at a definite place. (Maintaining our experience is not laborious, but only thinking [...].)”

“*Die Geistesmassen bedürfen zum Fortbestehen keines materiellen Trägers und üben auf die Erscheinungswelt keine dauernde Wirkung aus. Sie stehen daher in keiner Beziehung zu irgend einem Theile der Materie und haben daher keinen Sitz im Raume. Dagegen bedarf alles Eintreten, Entstehen, alle Bildung neuer Geistesmassen und alle Vereinigung derselben eines materiellen Trägers. Alles Denken geschieht daher an einem bestimmten Ort. (Nicht das Behalten unserer Erfahrung, nur das Denken strengt an [...].)*”

The relationships with the physical considerations will be clear. Here ‘inner manifold of the spiritual masses,’ there ‘inner condition of the aether stuff;’ here ‘material carrier required for thinking acts,’ there ‘aether stuff as the carrier for the action of gravitation and propagation of light;’ here ‘immaterial endurance of newly built representational masses,’ there ‘complete vanishing of the aether after its influx into the ponderable atoms;’ and so on.

From where did Riemann get his concept of ‘spiritual (or representational) masses’? It is well-documented (Scholz, 1982) that he, again, borrowed the idea from Herbart. In Herbart's writings the concept of a ‘representational *series*’ is of central importance in both philosophical and psychological respects. Here we only focus on the starting points out of which Herbart developed this concept. In-depth accounts may be found in, e.g., Murray & Bandemir (2002) or Banks (2005).

#### 4.2 Herbart's 'representational series', and the Problem of the Self

It is convenient at this point to go back to Herbart's epistemological reflections and his 'principles of psychology'; cf. HCW, p. 198 et seq. (Hereafter, the volume of Herbart's Collected Works containing the *Lehrbuch* and the first part of *Psychologie als Wissenschaft* is referred to as HCW.) Herbart resolutely rejects the common idea that it is the various faculties of the soul—such as thinking, feeling, desiring, remembering, etc.—that should constitute the subject of psychology. He rather holds that the principles of psychology are “those facts of the mind (*Bewusstsein*) from which the laws of what is happening within us can be discerned.” After some discussion of basic issues such as ‘Do such laws exist at all?’, ‘Is it possible, and how, to derive them?’—amongst that a striking query concerning the tension between the individual and the general: “What, then, if the individual wouldn't hold still enough to give way to a lawful abstraction?”—, Herbart proceeds asking two leading questions:

How do we come to know about the ‘facts of the mind’?—Answer: by way of introspection (*Selbstbeobachtung*).

What do we find?—Answer: the ‘representational series’ (*Vorstellungsreihe*), that is, a continual coming and going of representations.

‘Representation’ here can be anything that comes to or influences the mind, consciously or subconsciously, such as thoughts, perceptions, sensations, etc. The ‘fact of the mind’ highlighted in the concept of representational series then is the phenomenon that such representations are steadily popping up and suppressing each other in a ceaseless stream.<sup>8</sup> It is this phenomenon immediately accessible to introspection which psychology should focus on in the first place, rather than trying to set up a catalogue of distinct faculties of the soul. For, argues Herbart, those alleged faculties are in fact so interwoven that they cannot give rise to well distinguishable, clearly defined concepts on which psychology could be founded.

Herbart bases his concept of representational series not only on subjective experience, on the phenomena found by way of introspection. He also lends it a philosophical pillar in his reflections on the concept of the Self (*Ich*). The way he does so is of interest on its own, as it is downright intended, it seems, as a showpiece of his philosophy.

Herbart, who studied philosophy with Fichte, frequently recurs on his teacher despite (or because of) his openly ambiguous feelings. Accordingly, he starts praising Fichte for having tried to heave psychology beyond its status of being a plain collection of faculties of the soul: “he wanted to find not just the products of human mind, but the way they are produced” (*er wollte zu den Producten des menschlichen Geistes die Acte des Producirens finden*). Then, however, he blames Fichte for having only desired to do so without having accomplished anything; and goes on ridiculing Fichte's “gigantic project to deduce the world from the Self.” Nevertheless, in his analysis of the question ‘What is the Self?’ he starts out from Fichte's own definition:

“Self = The identity of subject and object”

Following his conception of philosophy as the “reworking of concepts” (*Bearbeitung der Begriffe*), he deliberately overwinds the conclusions from Fichte's definition in the desire to “make them collide with evident impossibilities; whence the concept of the Self [...] needs to be amended.” What is the evident impossibility? “The Self appears as a Given within the mind [...], but it lacks both an object and a subject, hence its whole matter.”

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<sup>8</sup> Accordingly, Herbart, in his physics-inspired mathematical psychology, conceives of the representations as *forces* acting against each other.

In order to demonstrate that with Fichte's definition the Self necessarily lacks an object, Herbart argues that the subject cannot be identical with any single of its predicates at the side of the objects. To that end he first characterizes the Self by its fundamental action: "The Self re-presents (i.e., sets itself before) Itself." (*Das Ich stellt Sich vor.*) That means, basically, that the Self—the subject for the moment being—permanently is looking into a mirror, where it finds—as its (temporary) object—Itself. But subject '=' object according to Fichte's definition, so one may substitute 'Itself' by 'Self' which, however, is nothing but that what 're-presents Itself'. Thus one gets: "The Self re-presents that what re-presents Itself." This argument can be reiterated, so it amounts to an infinite regress, to an "ever-lasting inquiry after Itself" that is never actually answered. Hence, the object eludes its determination, "is lacking."

The claim that the Self lacks a subject is demonstrated analogously, with the infinite regression extending in the other direction. Here the subject, 'Self', is successively being substituted by that what constitutes it, namely the act of re-presenting Itself. Herbart observes that the only way to escape the "double infinity into which the Self is expanding itself" would be to run into paradoxes such as "the deed shall coincide with the accomplished, [...], the act of re-presenting shall coincide with the picture itself." Hence, he concludes, the premise must be wrong, i.e., the determination of the Self as the identity of subject and object; QED. One sees, Herbart argues in the manner of a mathematical proof by contradiction.

After having taken down Fichte's definition of the Self, Herbart undertakes to amend it. His basic observation is that the object that the Self is setting before Itself may change—in fact, *has* to change—in each single re-presentation act. The Self is then identified with the flow, or series, of ideas, representations, perceptions, etc which the Self re-presents, or 'sets before Itself'—hence the 'representational series'. Those representations must continuously suppress each other and give way to different ones because otherwise, the Self would freeze to one particular of its representations, with which it cannot be identical: "The manifold representations have to annihilate each other if Selfness should be possible ([...] *wenn die Ichheit möglich sein soll*)." Cf. HCW, p. 286. The strain in the representational series, their excitability, provide the ground for the "forms we notice in our thought and contemplation."

Herbart emphasizes a particular property of "everything that is perceived inwardly," namely, that feelings, sensations, etc can get stronger and weaker. While objects, as such, cannot change, representations of objects can wane or become more vivid, hence can be assigned an intensity or magnitude. Herbart thus is led to consider magnitudes, or actually, proportions of and relationships between magnitudes and how these change (in lawful manner), as the fundamentals of his psychology. One will recall that essentially the same approach was adopted by Riemann who, too, wanted to infer the "coherence of the things" from the "observed coherence of the relations between magnitudes;" cf. Section 2.2. The circumstance that the representational series are amenable to a quantitative description lends itself to a more formal elaboration. In fact, Herbart's analysis of the Self is but the qualitative exposition of a strictly quantitative, mathematical development covering some 200 pages in his "Psychology as a Science."

#### **4.3 Herbart's 'Self' vs. Riemann's 'Agens'**

Did Riemann see a parallel between Herbart's representational series on one hand, and the dynamics of the aether stuff reflecting its inner form changes on the other hand? If the aether represents the substrate on whose state an action principle is acting upon, couldn't one see the representations, *Vorstellungen*, as the 'substrate' the Self is acting upon? This appears indeed possible considering that the representational series only stand for the phenomenal aspect of the Self. The phenomena "we notice in our thought and contemplation" first have to be brought about—which is achieved by nothing else than the Self through the *act* of re-presenting Itself. Or,

put slightly differently: the single representations build the elements of the representational series (the ‘Itself’) that are constantly being re-built by the Self. Thus the active and passive aspects of the Self belong to each other as “doing and coming about,” respectively (*Thun und Geschehen*; HCW, p. 471). Quite similarly in the *Neue Principien*, the *Stofftheilchen* build the elements of the aether whose inner state constantly evolves according to a minimum action principle. It thus would appear that the action principle and the Self represent, respectively, a physical and a psychological incarnation of the active aspect of Riemann’s Agens; moreover, that the passive aspect corresponds to the aether and the representations, respectively—or in fact: to the continually changing form or *state* of the aether and to the (relative) *intensities* of the representations (which Herbart took as the basis psychology is to be built on).

Of interest here is also a comparison with Herbart’s concept of the soul. In HCW, p. 108 et seq, Herbart characterizes the soul as a “simple entity” (*einfaches Wesen*) having no parts nor qualitative diversity. While “The simple What of the soul is entirely unknown, and will remain so forever,” that what matters, according to Herbart, are the relationships between distinct simple entities. These would alter each other (in regard to their “simple quality”), were it not the case that a simple entity resisted any perturbation of itself, thus striving to retain its simple quality. Internally, a simple entity is endowed with changing inner states the dynamics of which serves its self-conservation. “Such self-conservations singularly constitute what really happens in nature; and this makes for the connection of the events with the being.”

Herbart develops these ideas in a section entitled “Propositions (*Lehrsätze*) in metaphysics and natural philosophy,” touching subjects in which Riemann explicitly disagrees with Herbart; cf. footnote 3. That does not preclude, however, that Riemann started out from Herbart’s concept of the soul, then stripping off what appeared unpalatable or unnecessary to him—in best Herbartian manner. In fact, in Riemann’s introduction of the Agens there is a footnote emphasizing that it is sufficient to stick to the basic proposition “What an Agens strives to effectuate, has to be defined by the concept of the Agens;” and that one should *not* “[...] apply it [...] to entities (*Wesen*) allowing a manifold of determinations, but rather to real causes (*Realgründe*) of utmost simplicity” (RCMW, p. 524). Apparently, Riemann wanted to avoid the ontic connotations associated with *Wesen*, and instead proposed the Agens as a vantage point. Apart from that, the similarities between Riemann’s characterization of the Agens and Herbart’s concept of the soul are striking. On the other hand, Herbart’s ‘Soul’ and ‘Self’ are at least analagous in that they both give rise to the concept of representational series, which Herbart expressly declares as the “self-conservations of the soul” (HCW, p. 110, 289, 312). In fact, more than that: “The re-presenting subject is a simple substance, and is rightly called Soul” (HCW, p.289). Here is an attempt to summarize the observed relationships in a scheme.

	Agens		
Domain	active aspect	spiritual matter / passive aspect	matter
Physics	action principle	aether / state	ponderable atoms
Psychology	Soul / Self	representations / intensity	nerve cells

In words: The Agens as Riemann described it in the section *Causalität* is an evolving “state or proportion” (passive aspect) of ‘something’ re-acting upon itself (active aspect). For specific subject areas the ‘something’ admits a conceptual concretization: within the physical domain it is represented by the aether whose state evolves according to a mathematical (least) action principle; in psychology it corresponds to representations whose intensities change due to their

continual self-conservations, which Herbart downrightly identified with the Soul / Self. The substrates acted upon, aether and representations, not being plain-sense material stuff are tentatively dubbed 'spiritual matter'. (We here think of this notion in a general sense not restricted to the mental domain.) Finally, the appearances in the material world reflecting the aether dynamics and the representational series are 'implemented' on the basis of plain-sense material such as ponderable atoms or nerve cells (for the latter see next section).

The parallels between the physical and psychological domains become yet stronger when observing that Herbart does not distinguish between entities of a material or mental nature. Rather: "The difference between soul and matter does not pertain to the What of the entities, but is a difference in the manner we understand them." Thus, as far as what we apprehend of the self-conservations is manifested in space and time through physical properties such as elasticity, density, etc, we take them for phenomena of material entities; whereas, if our apprehension is manifested as ideas, desires, emotions, etc, we take them for expressions of the soul.

Interestingly, when looked at in this way, there is no essential difference left between soul and matter, all entities whatsoever sharing the same structural element that makes for their essence: their struggle for self-conservation in the presence of the other entities. With the difference between soul and matter levelled this way, Riemann's undertaking of putting the two of them on the same footing may appear less amazing. It even may have been Riemann's *response* to Herbart's challenge to search for the link between between mind and matter: *not* by focussing only on their respective appearances in the phenomenal world; but rather by going behind the phenomena and looking for the "simple and innerly ductile entities" giving rise to those appearances (HCW, p. 114). In passing, Herbart's conception of the "connection between soul and body" expressed in his *Lehrbuch* (HCW, p.114) is interesting and deserves a full quotation:

"The connection between mind and matter in the animals, particularly in humans, has much of the miraculous that has to be traced back to the sageness of Providence. However, it does not have it there where it is usually sought at first, because one considers the material as real inasmuch as it exists in space; and because one conceives of the human spirit as thinking, sensing, desiring by its nature: such that there is not any mediate link between the two of them. One should seek behind the material, as a spatial phenomenon, the simple and innerly ductile entities from which that phenomenon originates; one should consider the spirit as the representing soul; one should remember that the representations, as self-conservations of the soul, have to correspond to other self-conservations in other entities (in the elements of the nervous system, at first): thus one will understand that the chain of corresponding self-conservations might well extend still further, in fact to a whole system of entities that together represent One body; and one will no more consider it enigmatic that a sequence of inner states, without time course or spatial motion—such may occur as a concomitant phenomenon, however—, extends, forward and backward, from the tip of the foot to the brain and into the soul."

*"Die Verknüpfung zwischen Geist und Materie in den Thieren, insbesondere aber im Menschen, hat viel Wunderbares, das auf die Weisheit der Vorsehung muss zurückgeführt werden; aber sie hat es nicht da, wo man es zunächst zu suchen pflegt, weil man die Materie für real hält, sofern sie räumlich existirt; und weil man den menschlichen Geist als ein ursprüngliches Denken, Fühlen, Wollen betrachtet: so dass zwischen beiden jedes Mittelglied fehlt. Man suche hinter der Materie, als räumlicher Erscheinung, die einfachen und innerlich bildsamen Wesen, aus denen diese Erscheinung entspringt; man sehe den Geist an als die vorstellende Seele; man erinnere sich, dass den Vorstellungen, als Selbsterhaltungen der Seele, andre Selbsterhaltungen in anderen Wesen (zunächst in den Elementen des Nervensystems) entsprechen müssen: so wird man einsehn, dass die Kette zusammengehöriger Selbsterhaltungen*

*wohl noch weiter, dass sie durch ein ganzes System von Wesen, die sich zusammen als Ein Körper darstellen, fortlaufen könne; und man wird es nicht mehr räthselhaft finden, wenn von der Spitze des Fusses bis zum Gehirn und bis in die Seele eine Folge von innern Zuständen, ohne Zeitverlauf und ohne alle räumliche Bewegung,—dergleichen jedoch als begleitendes Phänomen vorkommen kann,—sich vorwärts und rückwärts erstreckt.”*

## 5. Physiology

Riemann's notes in the *Fragmente* that could be seen as relating to physiology are very scarce.<sup>9</sup> The basic idea is stated at the beginning of the first part entitled *Zur Psychologie und Metaphysik*: “With each simple act of thinking, something permanent, substantial enters our soul. [...] I call it ‘spiritual mass’ (*Geistesmasse*).” (See Section 4.1 for the full quotation.) In a footnote added to the statement of the OH (RCMW, p.529), Riemann credits the idea to Herbart: “[...] It is a consequence of Herbartian psychology that not the soul has substantiality, but each single representation built within us.” In fact, Herbart’s quantitative/mathematical theory seems to apply not so much to the representational series themselves, but, as it were, to the ‘products’ left behind by (or forming during) the self-conservation acts of the soul. Herbart calls these products ‘representational masses’ (*Vorstellungsmassen*), which then reappear in the *Fragmente* as ‘spiritual masses’ (*Geistesmassen*). Those representational/spiritual masses admit of certain basic operations; e.g., they may interact in essentially two ways with each other: depending on whether they pertain to entities of the same or of a different kind (e.g., sensations received through different modalities such as visual and auditory), such masses can “fuse” (*verschmelzen*) or build “complications” (*sie ‘compliciren sich’*). For more details of Herbart’s theory see Murray & Bandemir (2002).

Thus far Riemann sticks to Herbart. Further on, however, he considers it necessary to complement Herbart in regard to the nature and strength of the connections between the spiritual masses (subsequently abbreviated as ‘SpM(s)’, for convenience). Riemann suggests that every ‘newly arriving’ SpM—which “appears to us as a representation” (*Vorstellung*)—excites all other SpMs depending on the degree of their inner likeness: a SpM is excited the stronger, the less it differs from the “new” (just activated) one in regard to its “inner state (quality).” Thus SpMs may inhibit or reinforce each other in accordance with their similitude. Their totality may be considered as ‘the soul’: “The soul is a compact [...] and highly intertwined spiritual mass. It grows constantly through arriving spiritual masses, and this makes for its development.” Once built, SpMs are imperishable (*unvergänglich*), their interconnections endure as such; only their “relative strength [...] is modified by the accrual of new SpMs.” Their ontological status is described *ex negativo* (cf. the related citations quoted in Section 4.1): for persistence, SpMs do not need a material carrier; they do not interact with (physical) matter, nor do they reside anywhere in space. On the other hand, a material carrier is required for, and material processes go on during, the accrual and amalgamation of new SpMs. Hence, “all thinking occurs at a definite place.” Where is that place, and what is the carrier? According to Riemann the SpMs are built within the “cerebro-spinal system” by means of a chemical-electrical process conjoining the respective “active” sites. (Riemann here presumably refers to E. du Bois-Reymond’s

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<sup>9</sup> That does not mean that Riemann was only marginally interested in physiology. For example, in his last, unfinished paper “Mechanics of the ear” he picked up Helmholtz’s *Lehre von den Tonempfindungen* and entered into considerable anatomical and physiological detail. See also Laugwitz (1996, p. 280) and Smadja (2004).

demonstration, in the 1840ies, that electro-chemical processes are of primary importance in the activity of nerve cells.)

The term ‘spiritual mass’ sounds strange and may elicit reserve. Yet, some of the above speculations prompt associations with concepts of contemporary neuroscience. Thus Herbart’s and Riemann’s *Vorstellungs-* or *Geistesmassen* appear somehow related to the idea that objects are represented in the brain by means of cell assemblies, whose interactions are regulated, via chemical-electrical processes, by changes in the strengths of the synaptical connections. The immaterial character of “old” SpMs might be associated with the difficulty to pin down neuronal correlates of unactivated representations (which still should exist somehow).<sup>10</sup> The idea that material processes, on the other hand, should be involved in the development and integration of new representations/SpMs by means of “thinking acts” is reminiscent of the phenomenon of cortical plasticity: in connection with learning processes, for example, manifest brain-physiological effects are detectable, e.g., an increase of the number of synapses between neurons, and/or, of receptors within a synapse (e.g., Lynch, 2004). Of course, the recent findings of neuroscience were unavailable to Riemann (and Herbart), so drawing such parallels in hindsight is a moot attempt, ultimately.

What about Herbart’s stance in regard to the present topic? He undoubtedly felt the need to push his theory thus far as to meet with physiology. For instance, he reminds of “Locke’s right astonishment about the ‘narrowness of the human mind’,” which is capable of handling only a very small amount of the representations it actually disposes of, and raises the question of physiological constraints modifying this capacity (HCW, p. 343). Or he asks (HCW, p. 506) “What happens within me as I am thinking a, b, c, d [diverse *Vorstellungen*] aside to and distinct of each other?”—and goes on inquiring whether, and how, the affinity structure and serial ordering of these *Vorstellungen* might be represented in the brain,<sup>11</sup> ending with “*Such* are the questions that have to be answered.” Despite posing incisive and remarkably “modern” challenges, Herbart was skeptical about their being resolvable, and rather passed them on to his followers. For more information on these matters see Murray & Bandemir (2002); this article also contains a wealth of information about experimental results related to Herbart’s theory, along with continuative suggestions.

## 6. Closing Remarks

It cannot be said that Riemann’s bold unification attempt had a lasting impact. If at all, it was recognized by his fellow mathematicians with “awe and wonder,” or, in regard to his Other Hypothesis, with reluctance. Laugwitz (1996) points out that Euler, too, had tried to develop a similar unified theory of gravitation and light, the aether as a medium for planet movements and light propagation had already been discussed by Descartes and Huygens. The aether hypothesis

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<sup>10</sup> Another suggestion I owe to H. Primas, T. Filk, and J. Kornmeier (independently) is to translate ‘spiritual mass’ as ‘information’. This would go with the immaterial nature of the SpMs as well as with aspects of information production, transformation, transmission, and storing in the brain, thus could be an interesting idea to pursue.

<sup>11</sup> Herbart was not naïve on this point. While acknowledging that some of his speculations might be “ridiculous,” he clearly assumes that mental activity must somehow be implemented in the brain, *materially*. At the same time, he holds that the nervous system is a, miraculously purposeful, *servant* of the soul (but also can become a burden in the case of nervous diseases; HCW, p.116). To be sure, Herbart also simply is in error sometimes, e.g., when he rejects the idea that electro-chemical processes should be relevant to nerve physiology. Even here, however, he is lucid, adding “There could be something to it, and yet, the most important issues would remain unanswered; and in the end one mystery would be replaced by another one.” (HCW, p.113)

nowadays counts as refuted and has widely been abandoned. Of course, this does not interfere with the lasting fame of Riemann the mathematician.

Herbart, on the other hand, has been better remembered in pedagogy than as a philosopher and psychologist<sup>12</sup> (Scholz, 1982; e.g., there is—yet—no own entry in the Stanford Encyclopedia of Philosophy). The recent resurgence of interest in Herbart the philosopher is due to the increasing evidence for his indirect impact on the development of the natural sciences in the 19<sup>th</sup> century—indirect inasmuch it was his epistemology, his general philosophy of the tasks and methods of science, that were influential. This influence could be clearly demonstrated in the case of Bernhard Riemann (and has been further corroborated in this paper); see Scholz (1982, 1992). It would be interesting to know whether later developments such as Helmholtz’s sign and Hertz’s picture theory (Heidelberger, 1998; Patton, 2008) also bear traces of Herbart’s epistemology.

Yet, the idea of Herbart’s indirect impact via epistemology, essentially, might turn out incomplete. Thus in the *Fragmente* Riemann stucked closely also to Herbart’s considerations related to psychology and physiology—without achieving real progress, though. That might now change, however, due to the recent investigations of D. J. Murray and co-authors which point to possibly fruitful junctions between Herbart’s theories and challenges on one side, and contemporary psychophysics to neurophysiological learning theories on the other side. Is Herbart’s time to come yet?

Finally, it should be mentioned that there is a close vicinity in time between Riemann’s statement of the OH (“Found on 1<sup>st</sup> March 1853”) and the wide popularity in Germany of spiritualistic ideas and practices such as table-turning.<sup>13</sup> Did Riemann arrive at his hypothesis of a ‘spiritual substance built where stuff is disappearing’ guided by such ideas? This was indeed claimed by the astronomer of spiritualist twist, K. F. Zöllner (1834-1882), and approved of by C. Thiel even in the strong form that Riemann “conceived [his higher-dimensional geometry ...] with regard to the observed facts of spiritualism and so-called animal magnetism” (Thiel, 1987). Support for this view may be drawn, besides the OH itself, from another part of the *Fragmente* where Riemann, following G. Th. Fechner (1802-1887), sketches the idea of panpsychism (RCMW, pp. 511-518), and from a remark of E. Schering according to which Riemann once spoke to him of “[...] sites where the medium exits from the definite three-dimensional space to the several-dimensional space surrounding it everywhere” (Laugwitz, 1996, p. 278). Against that stands the well documentable influence of Herbart, particularly also on Riemann’s concept of spiritual masses—and spiritualism was alien to Herbart. Moreover, Riemann’s sobriety and his seriousness in religious matters as witnessed by, e.g., his friend Dedekind (RCMW, pp. 541-558) do not seem to go well with the views expressed by Zöllner or Thiel.<sup>14</sup>—More on this may be found in Laugwitz’s thorough exposition of Riemann’s philosophical ideas which also covers the historical context in great breadth (Laugwitz, 1996, pp. 271-284).

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<sup>12</sup> That may in part be due to the style of his writings which often read as if directed to students rather than to other scholars.

<sup>13</sup> See, e.g., the report of T. A. Hirst on a séance that took place on 24<sup>th</sup> April 1853 in an illustrious German evening party (Gardner & Wilson, 1993).

<sup>14</sup> Herbart was of course influenced by German idealism, if only by distancing himself from its protagonists (deriding, e.g., Kant’s and Schelling’s *Intellectuale Anschauung*). The unity of nature, the desire to look for single, fundamental principles, were central ideas of German *Frühromantik* and idealism that may well have reverberated in Riemann’s (and Herbart’s) speculations. Another impression that may, or may not, come to mind when reading Riemann’s *Fragmente* and records allowing a glimpse on his personality is that Riemann might have considered it a *religious duty* to try to trace the admirable expediency (*Zweckmäßigkeit*) of nature in its wholeness.

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## References

- Adámek, J., Herrlich, H., & Strecker, G. E. (1990). *Abstract and concrete categories*. New York: Wiley. Available online at <<http://katmat.math.uni-bremen.de/acc/acc.htm>>
- Banks, E. C. (2005). Kant, Herbart and Riemann. *Kant-Studien*, 96, 208-234
- Boudewijnse, G.-J.A., Murray D. J., & Bandomir, C. A. (1999). Herbart's mathematical psychology. *History of Psychology*, 2, 163-193
- Boudewijnse, G.-J. A., Murray D. J., & Bandomir, C. A. (2001). The fate of Herbart's mathematical psychology. *History of Psychology*, 4, 107-132
- Gardner, J. H., & Wilson, R. J. (1993). Thomas Archer Hirst – mathematician xtravagant. III. Göttingen and Berlin. *American Mathematical Monthly* 100, 619-625
- Heidelberger, M. (1998). From Helmholtz's philosophy of science to Hertz's picture-theory. In: Baird, D., Hughes, R.I.G., & Nordman, A. (eds.), *Heinrich Hertz: Classical Physicist, Modern Philosopher*. Dordrecht: Kluwer, pp. 9–24
- Herbart, J. F. (1850). *Lehrbuch zur Psychologie*. In: G. Hartenstein (ed.), *Johann Friedrich Herbart's Sämmtliche Werke*, Vol. 5 (1). Leipzig: Voss. [HCW]
- Herbart, J. F. (1850). *Psychologie als Wissenschaft, neu gegründet auf Erfahrung, Metaphysik und Mathematik. Erster, synthetischer Theil*. In: G. Hartenstein (ed.), *Johann Friedrich Herbart's Sämmtliche Werke*, Vol. 5 (1). Leipzig: Voss. [HCW]
- Hertz, H. (1894). *Die Prinzipien der Mechanik* (Gesammelte Werke, Vol. 3). Leipzig: Barth
- Laugwitz, D. (1996). *Bernhard Riemann 1826-1866: Wendepunkte in der Auffassung der Mathematik*. Basel: Birkhäuser
- Lynch, M. A. (2004). Long-term potentiation and memory. *Physiological Reviews*, 84, 87-136
- Murray D. J., & Bandomir, C. A. (2002). The predictive value of Herbart's mathematical psychology. *Psychologie et Histoire*, 3, 1-27
- Newton, I. (1988). *Mathematische Grundlagen der Naturphilosophie*. Sel., transl., introd., & ed. by E. Dellian. Hamburg: Meiner
- Patton, L. (2008). Hermann von Helmholtz. Entry in *Stanford Encyclopedia of Philosophy*. Available online <<http://plato.stanford.edu/entries/hermann-helmholtz/#ConEne184>>
- Primas, H. (2007). Non-Boolean descriptions for mind-matter problems. *Mind & Matter*, 5, 7-44
- Riemann, B. (1892). *Gesammelte mathematische Werke und wissenschaftlicher Nachlass*. Ed. by H. Weber & R. Dedekind. Leipzig: Teubner [RCMW]
- Scholz, E. (1982). Herbart's influence on Bernhard Riemann. *Historia Mathematica*, 9, 413-440
- Scholz, E. (1992). Riemann's vision of a new approach to geometry. In: L. Boi, D. Flament, J.-M. Saslanski (eds.), *1830 — 1930: A Century of Geometry. Epistemology, History and Mathematics*. Berlin: Springer, pp. 22–34
- Smadja, I. (2004). Équations aux dérivées partielles et philosophie naturelle. Remarques sur l'héritage herbartien de Bernhard Riemann. In: P. Valore (ed.), *Ars Experientiam Recte Intelligendi. Saggi filosofici*. Milan: Polimetrica, pp. 49-97

- Speiser, A. (1927). Naturphilosophische Untersuchungen von Euler und Riemann. *Journal für die reine und angewandte Mathematik*, 157, 105-114
- Thiel, C. (1987). Die Entmaterialisierung der Natur. In: Burrichter, C., Inhetveen, R., & Kötter, R. (eds.), *Zum Wandel des Naturverständnisses*. Paderborn: Schöningh, pp. 59-67
- Wackermann, J. (2008). Jenseits der psychophysischen Dualität: Wirklichkeit des Geistes. In: M. Peschl, A. Batthyany (eds.), *Geist als Ursache? Mentale Verursachung im interdisziplinären Diskurs*. Würzburg: Königshausen & Neumann, pp. 189–221