Weak Quantum Theory: Complementarity and Entanglement in Physics and Beyond

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Abstract

The concepts of complementarity and entanglement are considered with respect to their significance in and beyond physics. A formally generalized, weak version of quantum theory, more general than ordinary quantum theory of physical systems, is outlined and tentatively applied to two examples.

1 Introduction

Complementarity and entanglement are notions which have become popular through the significance they received in quantum theory. Nevertheless they were and are applied in other fields, even beyond physics, as well. There are cases in which their purely physical meaning is naturally extended, in other cases they are used in ways making the connnection to physics hard, inscrutable, or even impossible.

Applying complementarity and entanglement beyond physics, one is faced with three logical possibilities.

- Within a strong reductionist approach, one would understand every kind of complementarity and entanglement as a manifestation of the quantum theoretical significance of those notions in an apparently different context.
- Assuming that the formal scheme of ordinary quantum theory has realizations beyond physics, the application of complementarity and entanglement is possible by direct and complete analogy to ordinary quantum theory.
- A weaker assumption is that a generalized version of the formal scheme of ordinary quantum theory, in which particular features of ordinary quantum theory are not contained, should be used in some non-physical contexts. If concepts like complementarity and entanglement can still be defined in such a generalized scheme, a generalization of those notions beyond physics is achieved.

In this contribution we propose a formal way to define the concepts of complementarity and entanglement in the spirit of the third option. We do this in a manner allowing a stepwise relaxation of restricting conditions needed for their definition in the context of ordinary quantum physics. The main purpose of this approach is to sketch possibilities for applying the two notions in a less restricted, but equally precise way. As will be demonstrated, the intended areas of possible application beyond the scope of physics cannot be successfully addressed in terms of either of the first two options mentioned above. In other words, the attempt is to generalize the mathematical and conceptual framework of physical quantum theory in such a way that the generalized, weak version of the theory is still mathematically formulated, but no longer restricted to physics in its traditional scope.

To be a bit more explicit, complementarity will be extended beyond the concept of non-commuting properties of a quantum system such as momentum and position as elements of a C*-algebra. Entanglement, which is tightly related to complementarity, will similarly be extended beyond the concept of (generally) non-local correlations (not interactions) between non-commuting properties of quantum systems. In particular, a formal framework will be outlined that might facilitate using the concepts of complementarity and entanglement in situations exceeding the limits of physics as a science of the material world. For instance, the significance of complementarity and entanglement could be explored in philosophical, psychological or psychophysical problem areas, without loosing the desirable formal rigor and precision. In this context, the question is which features of physical quantum theory must be relaxed if one wants to apply weak versions of quantum theory to those problem areas.

The paper is organized in the following manner. In Sec. 2, we give some selected examples for complementarity and entanglement for physical, psychological, philosophical, and psychophysical situations. Sec. 3 provides a compact overview concerning the formal and conceptual framework of quantum theory. We use an algebraic point of view since the algebraic formulation is best suited for a clear and transparent discussion of conceptual issues. Moreover it is abstract enough to open the possibility that it can be applied beyond the physical domain. In Sec. 4, a weak version of quantum theory, neutral with respect to its application in a specific scientific field, will be presented. Conditions will be outlined under which weak quantum theory can be stepwise restricted in order to recover the ordinary quantum theory used in physics. Sec. 5 will indicate tentative applications, not yet worked out in final detail, of the weak version. The idea is to look for the concrete significance of general features of weak quantum theory in selected examples.

2 Complementarity and entanglement: some examples

Complementarity is a concept made popular by Bohr in his attempts to highlight crucial features of quantum theory. The textbooks by Meyer-Abich (1965), Murdoch (1987), and Pais (1991) provide a lot of details. A special issue of the journal "Dialectica", edited by Pauli (1948), contains articles on complementarity by leading physicists, including Bohr himself (Bohr 1948).

From a conceptual point of view, Bohr used the concept of complementarity to indicate a relationship between apparently opposing, contradictory notions which can be characterized in terms of a relationship of polarity. Complementary features typically exclude each other, but at the same time complement each other mutually to give a complete view of the phenomenon under study. This is nicely demonstrated by the design Bohr once selected for a medal with which he was honored: it shows the text "contraria sunt complementa", accompanied by the Chinese Yin-Yang symbol.

The various examples which Bohr discussed as complementary over the years are of different significance and status. To study the corresponding differences in detail, it is necessary to look somewhat closer at various "complementary" pairs of notions. In particular, it is worthwhile to explore which ones among them are definitely related to entanglement, which ones are definitely not related to entanglement, and which ones are (presently) not understood well enough to draw this distinction clearly.

The best formalized examples of complementary pairs of notions are those referring to pairs of non-commutative properties of a system, so-called observables. Well-known examples are position Q and momentum P (with a generally continuous spectrum) or spins in different directions (with only two discrete eigenvalues). The fact that, for instance, P and Q do not commute is formally expressed as a Heisenberg type commutation relation

$$[P, Q] = PQ - QP = i\hbar \mathbb{1}, \tag{1}$$

where $\hbar$ is the Planck action h divided by 2π. For more details see, e.g., Jammer (1974) and references given there.

The non-commutativity or incompatibility of observables is at the heart of the non-Boolean structure of quantum theory, and as such it is a major precondition for situations in which states of systems are entangled. Entanglement characterizes the fact that a system in a pure state in general cannot be simply decomposed into subsystems with pure states. In a certain sense, such subsystems do not exist a priori but must be generated by appropriate procedures. This has conceptual consequences first pointed out by Einstein et al. (1935); the term entanglement itself was coined by Schrödinger (1935).

The theoretical arguments (Bell 1964) and experimental results (Aspect et al. 1982) which, beyond any reasonable doubt, confirmed the entangled (holistic) characteristics of quantum systems were based on spin-1/2 systems, i.e. spin measurements on photons. The crucial empirical feature in this context are so-called nonlocal (holistic) correlations between two photons. Popular misconceptions notwithstanding, it is illegitimate to interpret these correlations due to causal interactions between the photons.

In particular situations, the relationship between energy and time can also be considered as complementary in the sense of non-commutative observables. Although traditional quantum theory was not general enough to enable a formal incorporation of corresponding observables, later developments have shown that energy and time (and related observables) can be rigorously treated as non-commutative in a more general framework. Major progress in this respect has been achieved in the theory of stochastic and ergodic systems (Tjøstheim 1976, Gustafson and Misra (1976), Misra (1978).

If observables are non-commutative, this implies uncertainty relations (such as Heisenberg's uncertainty relations) between them. For position and momentum, Heisenberg's uncertainty relation reads:

$$\Delta p \Delta q \geq \hbar/2 \tag{2}$$

The meaning of this relation is that a quantum system cannot be in a state in which both P and Q have dispersion-free (definite) expectation values. The origin of relation (2) is thus of ontic character and goes beyond epistemic problems of measurement errors or computation errors.

In ordinary quantum theory, P and Q are maximally incompatible in the sense that for every eigenstate of Q all values of P are equally probable and vice versa. In our approach, we shall call two observables complementary whenever they do not commute, even if they are only incompatible rather than maximally incompatible. In quantum theory, this means that there are pure eigenstates of one of the observables which are not eigenstates of the other one. It will turn out that this more general notion of complementarity is suitable for generalizations beyond physics.

The existence of an uncertainty relation does not necessarily imply that it originates from non-commutative quantum observables. There are classical observables whose uncertainty relations are simply due to Fourier reciprocity. For instance, engineers have known for long that the classical bandwidth $\Delta\omega$ (energy) and the classical duration Δt (time) of a signal satisfy an uncertainty relation

$$\Delta\omega\Delta t \geq 1/2 \tag{3}$$

which is not restricted to ordinary quantum systems and does therefore not imply ordinary quantum entanglement. This situation will be discussed in Sec. 5.1, see particularly the context of Eq. (41).

In addition to non-commutative properties of physical systems, it is also possible to formalize particular kinds of descriptions of physical systems in a non-commutative manner. For instance, one can show that a description of the temporal evolution of a system in terms of a Liouville operator L and an information theoretical description of the same system in terms of an information operator M are complementary in the sense that

$$[L, M] = LM - ML = iK\mathbb{1} \tag{4}$$

where K is the (dynamical) Kolmogorov-Sinai entropy of the system (Atmanspacher and Scheingraber 1987).

Such a complementarity of dynamical descriptions is a special case within the broader class of deterministic versus statistical descriptions, whose distinction can be related to that of ontic versus epistemic descriptions (Scheibe 1973, Primas 1990). In contrast to corresponding formalized kinds of complementarity, there are other pairs of descriptions whose "complementarity" is not formally backed up to a comparable degree. A historical example in quantum physics is the relationship between wave-oriented and particle-oriented descriptions. Furthermore, beyond the limits of physics, there are complementarities of substance and form, of allopoietic and autopoietic systems, of statistical and deterministic descriptions, of efficient and final causation, and many others. For all of them, it would be as difficult as interesting to show whether they are related to some kind of entanglement.

Leaving the natural sciences, things become even more complicated, and formal approaches are, at least at present, totally lacking. Nevertheless, e.g., the cognitive sciences and psychology offer a multitude of examples which might refer to complementarity. The relationship between conscious and unconscious processes (Jung 1971, Pauli 1954), between Jung's psychological types (thinking and feeling, intuition and sensation; Jung 1921), between substantive and transitive mental states (James 1950), between bi- or multistable states of perception (Plaum 1992, Kruse and Stadler 1995) and between multiple personalities (Jordan 1947) are all candidates for complementary relations which call for more detailed investigation.

Bohr considered the concept of complementarity as so fundamental and widely applicable that he even used it to characterize philosophical or philosophy-related problems (cf. Bernays 1948). Three often quoted examples refer to the definition versus the usage of terms, clarity versus truth, and goodness versus justice. The complementary pair of confirmation and novelty has been proposed for a suitable definition of meaning in terms of pragmatic information (Weizsäcker 1974, Kornwachs and Lucadou 1985). Probably one of the most far-reaching applications of complementarity, however, concerns the relation between mind and matter or, respectively, between mental and material observables of systems.

One crucial problem area in this respect is the relationship between the psychological experience of mental activity and the (neuro-)physiological brain processes without which such experience does (most likely) not exist (cf. Chalmers 1996). This example shows in a particularly clear manner how problematic it is to prematurely interpret such relationships in terms of causality. The basic concept for a complementary relationship between mind and matter is that of correlations, i.e., "neural correlates of consciousness", between the considered notions. The question whether there are causal interactions on top of these correlations is, of course, important. However, assigning causality too quickly can lead to ill-posed questions. It is illustrative to consider many features of the debate between adherents of "mind over matter" versus those of "matter over mind" from this point of view.

Another, even more speculative, approach to the relationship between mind and matter was put forward by Jung and Pauli (1952). Inspired by the old philosophical concept of psychophysical parallelism, such as in Leibniz's philosophy, Jung and Pauli explored the idea of a complementary relationship between mind and matter in a very broad sense. An essential aspect of their speculations was a reality behind (or beyond) those two realms which is, e.g. for epistemological purposes, split into a mental and a material domain. This split (sometimes called Cartesian cut) destroys the primordial wholeness of the background reality, and "synchronistic" correlations between mind and matter remain as remnants of the lost wholeness. Such a scenario is obviously inspired by the quantum theoretical conception of entanglement (see, e.g., Atmanspacher and Primas 1996, Walach and Römer 2000, Atmanspacher 2001). Concrete and detailed indications concerning the substance of such a scenario are, at least to our knowledge, not available so far.

In view of all these examples, the question arises whether it is possible to generalize the standard quantum theoretical framework in such a way that complementarity and entanglement might be useful concepts in a broader context. Moreover, since the two concepts are not identical with each other, it is worthwhile to formally explore the conditions which particular situations must satisfy to allow us to talk about complementarity and entanglement. For this purpose, the next section gives a brief outline of the essential formal and conceptual features of standard quantum theory from an algebraic perspective. Subsequently, it will be studied which of those features can be relaxed without losing the structures necessary for complementarity and entanglement.

3 Algebraic quantum theory in a nutshell

The algebraic formulation of quantum theory is the most appropriate framework for discussing its formal and conceptual structure and possible generalizations. In this section, we give a brief overview of algebraic quantum theory in order to provide a solid foundation for the following sections. More comprehensive and detailed accounts can be found in standard textbooks and monographs, such as Haag (1996), Piron (1976), Primas (1983), or Thirring (1981).

The fundamental notions for the description of a quantum system Σ are those of *observables* and *states*. An observable is any property of the system Σ which can – at least in principle – be measured in a reproducible way. To every observable A belongs a set $specA \subset \mathbb{C}$ of complex numbers, the set of possible results of a measurement of A. The observables of Σ generate a C^*-algebra $\mathcal{A}$ which is called the *observable algebra* of Σ. The system Σ can be in different physical states, and a physical state z determines the probability distributions for the measured values of any observable A. In quantum theory, states are positive linear functionals on $\mathcal{A}$.

Let us explain these notions in more detail. First of all, the observable C^*-algebra $\mathcal{A}$ is an algebra over the complex numbers $\mathbb{C}$, which means that addition and multiplication of elements in $\mathcal{A}$ as well as multiplication with complex numbers are defined such that for $A, B, C \in \mathcal{A}, \alpha, \beta \in \mathbb{C}$:

A1 $A + (B + C) = (A + B) + C$

A2 There is a zero element 0 with $0 + A = A + 0 = 0$.

A3 To every A there is an opposite element $-A$ such that $A+(-A) = A-A = 0$.

A4 $A + B = B + A$

A5 $1 \cdot A = A$

A6 $\alpha(\beta A) = (\alpha\beta)A$

A7 $(\alpha + \beta)A = \alpha A + \beta A$

A8 $\alpha(A + B) = \alpha A + \alpha B$

A9 $A(BC) = (AB)C$

A10 There is a neutral element $\mathbb{1} \in \mathcal{A}$ with $\mathbb{1}A = A\mathbb{1} = A$.

A11 $(\alpha A)B = A(\alpha B) = \alpha AB$

A12 $A(B+C) = AB + AC, (B+C)A = BA + CA$

Moreover, $\mathcal{A}$ is a star-algebra, i.e., there is an involution $A \mapsto A^*$ with:

S1 $(\alpha A + \beta B)^* = \overline{\alpha} A^* + \overline{\beta} B^*$ ($\overline{\alpha}$ is the complex conjugate of $\alpha \in \mathbb{C}$)

S2 $(AB)^* = B^* A^*$

S3 $(A^*)^* = A$

$\mathcal{A}$ is also a *Banach star-algebra*, which means that there is a norm function $A \mapsto ||A|| \in \mathbb{R}$ with

B1 $||A|| \geq 0,\ ||A|| = 0 \ \Leftrightarrow A = 0,\ ||\alpha A|| = |\alpha|\ ||A||$

B2 $||A+B|| \ \leq \ ||A|| + ||B||$

B3 $||AB|| \ \leq \ ||A||\ ||B||$

B4 $||A^*|| \ = \ ||A||$

B5 $\mathcal{A}$ is complete with respect to the norm $|| \cdot ||$, i.e., every sequence A_n with $||A_m - A_n|| < \varepsilon$ for $m, n \geq N(\varepsilon)$ converges to a (unique) element of $\mathcal{A}$.

Finally, the so-called C^**-condition* for $\mathcal{A}$ means:

C1 $||A^* A|| = ||A||^2$ (B4 follows from C1)

A state z is a (continuous real) functional $A \mapsto z(A) \in \mathbb{C}$ on $\mathcal{A}$ with:

Z1 $z(\alpha A + \beta B) = \alpha z(A) + \beta z(B), \quad z(A^*) = \overline{z(A)}$

Z2 $z(A^* A) \geq 0$

The reader will notice that we also admit an "impossible" zero state $z = 0$. The set of all states is *convex*, hence with z_1 and z_2 also

$$z = \alpha z_1 + (1-\alpha) z_2, \quad 0 \leq \alpha \leq 1 \tag{5}$$

is a state. A state z is called *pure*, if it does not admit a non-trivial decomposition of type (5). Pure states contain maximal information about the system Σ.

For every state $z \neq 0$ we can define *expectation values* of observables A:

$$E_z(A) = \frac{z(A)}{z(\mathbb{1})}. \tag{6}$$

$E_z(A)$ is the mean of measured values of A in the state z, if the (reproducible) measurement of A is repeated many times in the same state z.

In quantum theory, *specA*, the set of possible measured values of A is given by those $\alpha \in \mathbb{C}$, for which $(A - \alpha\mathbb{1})$ has no inverse in $\mathcal{A}$. Moreover, only self-adjoint elements of $\mathcal{A}$ are normally admitted as observables:

$$A^* = A; \tag{7}$$

The *uncertainty* σ_A^z of a self-adjoint observable $A \in \mathcal{A}$ in a state $z \neq 0$ is defined as

$$(\sigma_A^z)^2 = E_z((A - E_z(A))^2) = E_z(A^2) - E_z(A)^2 \geq 0. \tag{8}$$

One can derive a general uncertainty relation

$$\sigma_A^z \sigma_B^z \geq \frac{1}{2} \mid E_z(AB - BA) \mid. \tag{9}$$

In quantum theory, as opposed to classical theory, the observable algebra is not commutative. This means that, in general, $AB - BA \neq 0$, and there is no state in which all uncertainties vanish. Commuting observables with $AB = BA$ are called *compatible*, non-commuting observables with $AB \neq BA$ are called *incompatible* or *complementary.*

Propositions are special observables, whose measured values can only be "yes" $\widehat{=}\ 1$ or "no" $\widehat{=}\ 0$. They are given by elements $P \in \mathcal{A}$ with

$$P^* = P,\ P^2 = P. \tag{10}$$

For $z \neq 0$, $E_z(P)$ is the probability that P is measured as "yes" in the state z.

To every proposition P we can associate a *negation* $\overline{P} = \mathbb{1} - P$ with

$$E_z(\overline{P}) = 1 - E_z(P). \tag{11}$$

The *conjunction* of two propositions P_1 and P_2 is given by the proposition

$$P_1 \wedge P_2 = \lim_{n\to\infty} (P_1 P_2)^n, \tag{12}$$

and the *adjunction* is defined as

$$P_1 \vee P_2 = \mathbb{1} - (\mathbb{1} - P_1) \wedge (\mathbb{1} - P_2) = \overline{\overline{P}_1 \wedge \overline{P}_2}. \tag{13}$$

We find

$$\begin{aligned} P_{1,2}(P_1 \wedge P_2) = (P_1 \wedge P_2)P_{1,2} = P_1 \wedge P_2, \\ P_{1,2}(P_1 \vee P_2) = (P_1 \vee P_2)P_{1,2} = P_{1,2}. \end{aligned} \tag{14}$$

For compatible P_1, P_2 we simply have

$$\begin{aligned} P_1 \wedge P_2 &= P_1 P_2 \\ P_1 \vee P_2 &= P_1 + P_2 - P_1 P_2 \end{aligned} \tag{15}$$

The *spectral theorem* states that every self-adjoint observable A can be equivalently represented as the adjunction of propositions which are mutually compatible and compatible with A.

In the usual formulation of quantum theory, observables are operators and states are density matrices on a *Hilbert space* $\mathcal{H}$. In the (more general) algebraic formulation, such Hilbert space formulations can be recovered by the so-called *GNS-construction*.

Every state $z \neq 0$ defines a vanishing ideal

$$I_z = \{C \in \mathcal{A} \mid z(C^*C) = 0\}. \tag{16}$$

The quotient algebra $\mathcal{A}/I_z$, together with the scalar product

$$< [A], [B] >_z = z(A^*B) \tag{17}$$

on the equivalence classes $[A] = A + I_z, [B] = B + I_z$, then gives rise to a Hilbert space $\mathcal{H}_z$ and to a representation

$$A[B] = [AB] \tag{18}$$

of $\mathcal{A}$ on $\mathcal{H}_z$.

For what follows it is important that every observable $A \in \mathcal{A}$ acts on the set Z of all states: to every state $z \in Z$ and every observable $A \in \mathcal{A}$ we can associate a state

$$\rho_A(z) \tag{19}$$

defined by

$$(\rho_A(z))(B) = z(A^*BA). \tag{20}$$

Obviously,

$$\rho_{A_1A_2}(z) = \rho_{A_1}\rho_{A_2}(z). \tag{21}$$

We can identify A and ρ_A up to a complex phase, which is generically fixed, if *spec*A is known. The action of observables on states corresponds to the active interpretation of observables as operations changing the state of a system. For a proposition P and a state z with $z(P) \neq 0, \rho_P(z) =: Pz$ is a state in which P is true with certainty:

$$E_{Pz}(P) = \frac{Pz(P)}{Pz(\mathbb{1})} = \frac{z(P^3)}{z(P^2)} = 1. \tag{22}$$

Application of P corresponds to verification of P. One remarkable feature of quantum theory is its *holistic* character. If a quantum system Σ is composed of two subsystems Σ_1 and Σ_2, then the state of Σ is in general not determined by the states of Σ_1 and Σ_2. The reason for this resides in the non-commutativity of the algebra $\mathcal{A}$ of Σ. $\mathcal{A}$ contains subalgebras $\mathcal{A}_1$ and $\mathcal{A}_2$ refering to the subsystems Σ_1 and Σ_2. If Σ_1 and Σ_2 are well-separated, $\mathcal{A}_1$ will commute with $\mathcal{A}_2$. Nevertheless, there will be observables $A \in \mathcal{A}$ of the total system which are incompatible with observables in $\mathcal{A}_1$ and/or $\mathcal{A}_2$. Intimately related to this, there exist *entangled states* of the total system, in which the values of observables in $\mathcal{A}_1$ and $\mathcal{A}_2$ are undetermined but correlated without any interaction between Σ_1 and Σ_2. These correlations cannot be used for transmitting information between Σ_1 and Σ_2. If, in addition, the algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ are non-commutative, *Bell's inequalities* for correlations between measured values of observables for Σ_1 and Σ_2, which must be satisfied under the hypothesis of local realism, can be shown to be violated. This is the case in quantum theory.

Under the assumption that there is no instant interaction-at-a-distance between Σ_1 and Σ_2, the (experimentally observed) violation of Bell's inequalities means that a realistic interpretation, to the effect that the outcome of measurements on Σ_1 and Σ_2 is predetermined by objective features such as local hidden parameters already before measurement, is excluded. The indeterminacy of quantum theory is not *epistemic*, i.e. due to incomplete knowledge or inevitable perturbations of the state of the quantum system Σ, but *ontic*. This fact can and should be interpreted as a consequence of the holistic character of quantum theory.

Separation and composition are problematic operations in complex systems. Quite a basic example is the separation given by the *epistemic splitting* of a physical system into an observing and an observed subsystem. A quantum theoretical analysis of the measuring process reveals that the stochastic character of quantum theory can be attributed to a neglect of holistic correlations between observing and observed subsystem if the system is in an entangled state. Entangled states of complex systems have a tendency to evolve into decoherent states which are effectively indistinguishable from an incoherent superposition of separable states.

4 Weak quantum theory

In this section, we outline some assumptions under which particular features of quantum theory can be generalized to a framework broader than that of ordinary quantum theory. For such a framework, we expect the notions of systems, observables and states to remain valid and meaningful.

A system Σ is considered as a part of reality in a very general sense, i.e. it can be the object of attention and investigation beyond the realm of ordinary quantum theory, possibly even beyond the limitations set by the concept of a material reality. Even though the isolation of parts of reality is expected to be a problematic operation, its possibility, at least in some approximate sense, is the prerequisite for any act of cognition and, in fact, already implicit in the epistemic split between subjects and objects of cognition.

An observable is any (more or less) meaningful property of the system Σ which can be investigated in a given context. Non-trivial observables must exist, whenever Σ has enough internal structure to be a possible object of a meaningful study. To every observable A there should belong a set *specA* of possible results of an investigation of A. In the general case addressed in this section, the relation between A and *specA* will be different from the quantum theoretical situation described in the previous section.

It must at least be conceivable that the system Σ exists in different states. Different states should reflect themselves in different outcomes of observations associated to observables A. Even if the system Σ de facto always is in the same state z, it must be possible to conceive it in other states. (Otherwise, nothing could be learned about Σ.) The possibility of different states is indispensible for discussing stability criteria for the system Σ, which has to maintain its identity under "unsubstantial" changes.

In addition, the notion of a state also has epistemic aspects, reflecting various degrees of knowledge about Σ. As in ordinary quantum theory, we call a state pure if it contains maximal information about Σ. Normally, the state of the system Σ will change in the course of time, under the influence of other systems and as a consequence of being observed and investigated.

As in ordinary quantum theory, we associate a set $\mathcal{A}$ of observables and a set Z of states to every system Σ. Here $\mathcal{A}$ and Z are meant to be sets in a naive sense, not necessary in the sense of axiomatic set theory. Our task will be to investigate the general structures of $\mathcal{A}$ and Z.

The first property of $\mathcal{A}$ we want to formulate is

Axiom I: To every observable $A \in \mathcal{A}$ belongs a set *specA*, the set of possible outcomes of a "measurement" of A.

In the previous section we saw that quantum observables can be identified with functions $A : Z \to Z$ on the set of states. This fact, which underlines the active, operational character of observations, should be valid more generally. We thus formulate

Axiom II: Observables are (identifyable with) *mappings* $A : Z \to Z$, which associate to every state z another state $A(z)$.

Axiom II implies that observables can be composed as maps on Z, where the map AB is defined by first applying B and then A. We shall assume

Axiom III: With A and B, also AB is an observable.

A direct consequence of Axiom III is the *associativity* of the composition of observables:

$$A(BC) = (AB)C \tag{23}$$

Moreover, we can postulate

Axiom IV: There is a unit observable $\mathbb{1}$ such that $\mathbb{1}A = A\mathbb{1} = A \ \forall \ A \in \mathcal{A}$.

$\mathbb{1}$ is the operation on Z which does not change any state, it corresponds to a proposition which is always true, so $spec\mathbb{1} = \{\text{true}\}$. Axioms II–IV mean that the set of observables has the structure of a *monoid*, which is also called a *semigroup with unity* or an *associative magma with unity*.

For formal completeness we also need an "impossible" *zero state* $z = o$ and a *zero observable* 0 with $spec0 = \{\text{false}\}$, which corresponds to an always false proposition.

Axiom V: There are a zero state o and a zero observable 0 such that

$$\begin{aligned} 0(z) &= o \ \forall \ z \in Z, \\ A(o) &= o \ \forall \ A \in \mathcal{A}, \\ A0 = 0A &= 0 \ \forall \ A \in \mathcal{A}. \end{aligned} \tag{24}$$

One may wonder about the *addition* of observables which, after all, plays an important role in ordinary quantum theory. But even there, the operational definition of $A+B$ is problematic; for instance there is no general strategy to construct a measuring device for $A+B$. Nevertheless it should be mentioned that Jordan algebras, an early attempt to generalize ordinary quantum theory from non-commutative, associative algebras to commutative, non-associative algebras, are explicitly constructed on the basis of an addition of observables (for more details see Primas 1983).

In the generalized framework addressed here, there is no evident place for the addition of observables. As a consequence, the set of states in weak quantum theory cannot be presupposed to be convex as in ordinary quantum theory. This relates to the fact that a probability interpretation is not feasible within the general framework of weak quantum theory (but can be implemented by restricting its generality, see below).

There is no reason to assume commutativity, $AB = BA$, for all $A, B \in \mathcal{A}$. Rather there will be both commutative (compatible) and non-commutative (incompatible) pairs of observables, depending on whether $AB = BA$ or $AB \neq BA$. This means that the monoid structure of $\mathcal{A}$, however poor and general, contains complementarity and entanglement as essential features of quantum theory.

A simple model obeying axioms II–IV can be obtained by identifying Z with the states of a rigid body in space and $\mathcal{A}$ with the set of motions. This indicates that additional structure is required for a reasonable generalization of ordinary quantum theory. Such additional structure is, indeed, at hand, because there are *propositions* P among the observables $\mathcal{A}$ which play a distinguished role. A proposition $P \neq 0, \mathbb{1}$ is an observable whose outcome is either true or false:

$$specP = \{\text{true}, \text{false}\} \text{ for } P \neq 0, \mathbb{1} \tag{25}$$

Moreover, to every proposition P there must be a negation $\overline{P}$, which gives "false" if and only if P gives "true".

We now give a few rather evident axioms assumed to hold for propositions.

Axiom VIa:

$$\begin{aligned} P^2 &= P, \\ \overline{\overline{P}} &= P, \ \overline{\mathbb{1}} = 0, \\ P\overline{P} &= \overline{P}P = 0. \end{aligned} \tag{26}$$

For *compatible* propositions $P_1, P_2, P_1P_2 = P_2P_1$ we can define a *conjunction*

$$P_1 \wedge P_2 = P_2 \wedge P_1 = P_1P_2 \tag{27}$$

and an *adjunction*

$$P_1 \vee P_2 = \overline{\overline{P}_1\overline{P}_2} = P_2 \vee P_1 \tag{28}$$

with the properties

$$\begin{aligned} P \wedge P = P \vee P &= P, \\ P_1 \wedge (P_2 \wedge P_3) &= (P_1 \wedge P_2) \wedge P_3, \\ P_1 \vee (P_2 \vee P_3) &= (P_1 \vee P_2) \vee P_3, \\ P_1 \wedge (P_1 \wedge P_2) = (P_1 \wedge P_2) \wedge P_1 &= P_1 \wedge P_2, \\ P_1 \vee (P_1 \vee P_2) = (P_1 \vee P_2) \vee P_1 &= P_1 \vee P_2, \\ 0 \wedge P_1 = P_1 \wedge 0 &= 0, \\ \mathbb{1} \wedge P_1 = P_1 \wedge \mathbb{1} &= P_1, \\ 0 \vee P_1 = P_1 \vee 0 &= P_1, \\ \mathbb{1} \vee P_1 = P_1 \vee \mathbb{1} &= \mathbb{1}. \end{aligned} \tag{29}$$

Moreover, we postulate the meaning of P as verification.

Axiom VIb: If $P(z) \neq o$, then $P(z)$ is a state in which P is true with certainty.

Finally, we formulate an axiom replacing the spectral theorem of ordinary quantum theory. Every observable A should be equivalent to a set of mutually exclusive propositions. More precisely, let A be an observable and $\alpha \in specA$. A_α denotes the proposition that the outcome of a measurement of A is $\alpha \in specA$. Then we have

Axiom VIc:

$$A_\alpha A_\beta = A_\beta A_\alpha = 0 \text{ for } \alpha \neq \beta, \quad AA_\alpha = A_\alpha A, \quad \bigvee_{\alpha \in specA} A_\alpha = \mathbb{1}. \tag{30}$$

A and B are compatible if and only if A_α and B_β are compatible for all $\alpha \in specA$ and $\beta \in specB$.

In general, incompatible observables do not have simultaneous definite values.

Although the generalized *weak quantum theory* as defined by axioms I–VI is considerably weaker than ordinary quantum theory, they share the following two characteristic features.

- Incompatibility and complementarity arise due to the non-commutativity of the multiplication of observables.
- Holistic correlations and entanglement arise if for a composite system observables pertaining to the whole system are incompatible with observables of its parts.

In the latter context, it should be emphasized that weak quantum theory itself refers to the description of the system as a whole. Any identification of parts or subsystems implies a specific choice of representation in terms of partial monoids. This choice remains open in the general framework of weak quantum theory. In weak quantum theory, the absence of a vector space structure implies that there is no tensor product construction for the set of observables of a composite system. In general, we can only expect:

$$\mathcal{A} \supset \mathcal{A}_1 \times \mathcal{A}_2,\ Z \supset Z_1 \times Z_2, \tag{31}$$

$$\mathcal{A}_1(Z_1) \subset Z_1,\ \mathcal{A}_2(Z_2) \subset Z_2. \tag{32}$$

A similar remark applies to the specific form of the dynamical evolution of (sub-)systems in weak quantum theory. The dynamics of a system is generally described by a one-parameter (semi-)group of endomorphisms. The process generating subsystems (e.g., by measurement) and the dynamics of interacting subsystems depends on details of the considered system and its decomposition.

There are other features of weak quantum theory which are not shared by ordinary quantum theory.

- There is no quantity like Planck' s constant h which in ordinary quantum theory quantifies the degree of non-commutativity of two given observables. This indicates that in the generalized, weak theory, complementarity and entanglement are not restricted to a particular degree of non-commutativity as it is the case for ordinary quantum mechanics.
- Since the addition of observables is not defined in the general framework of weak quantum theory, there is no convex set of states, there are no linear expectation value functionals, and there is no probability interpretation. Probability distributions on the sets $specA$ do not occur and are not calculable in weak quantum theory. As a matter of fact, the mere concept of probability will be absent in many situations (e.g., in an exploration of a work of fine art or of the intensity of an emotion).

- There is no way to generalize Bell's inequalities up to the general framework of weak quantum theory, and there is no way to argue that complementarity and indeterminacy in weak quantum theory are of ontic rather than epistemic nature. On the contrary, one would expect them to be of rather innocent epistemic origin in many cases, for instance, due to incomplete knowledge of the system or uncontrollable perturbations by observation.

Axioms I–VI, characterizing weak quantum theory, can be regarded as minimal requirements for a meaningful general theory of observables and states of systems. Between the weak version of quantum theory and its ordinary version, there are intermediate theories which can be obtained by enriching the axioms stepwise. Let us first discuss enrichments of the propositional axiom VI. Subsequently we shall add a probability interpretation of states.

One evident option is to postulate that the conjunction and adjunction of propositions is also defined in the less intuitive case of *incompatible* P_1 and P_2 such that propositions $P_1 \wedge P_2$ and $P_1 \vee P_2 = \overline{\overline{P}_1 \wedge \overline{P}_2}$ always fulfil the conditions of equations (29). In addition, it is natural to postulate

$$\begin{aligned} P_1 \wedge (P_1 \vee P_2) &= (P_1 \vee P_2) \wedge P_1 = P_1, \\ P_1 \vee (P_1 \wedge P_2) &= (P_1 \wedge P_2) \vee P_1 = P_1 \wedge P_2. \end{aligned} \tag{33}$$

The stronger distributivity condition

$$\begin{aligned} P_1 \wedge (P_2 \vee P_3) &= (P_1 \wedge P_2) \vee (P_1 \wedge P_3), \\ P_1 \vee (P_2 \wedge P_3) &= (P_1 \vee P_2) \wedge (P_1 \vee P_3), \end{aligned} \tag{34}$$

is not even satisfied in ordinary quantum theory. If every propositional subsystem generated by two compatible propositions with $P_1 \wedge P_2 = P_1$ and their negations is Boolean, then (modulo some technical complications) the propositional system is already isomorphic to a system of orthogonal projectors in a Hilbert space (Piron 1976, Thirring 1981). This Boolean property does not follow from axioms I – VI.

It cannot be prescribed in general which of these additional assumptions are applicable in a concrete situation. It will be unavoidable to consider details of the given context for corresponding decisions.

For a probability interpretation of states, one does not loose much by assuming $specA \in \mathbb{C}$, because it is very plausible that the set of outcomes of A can be mapped into the complex numbers in a one-to-one way. Assuming this, the existence of a probability interpretation amounts to postulating for every $z \neq 0$ the existence of an expectation value functional

$$\begin{aligned} E_z : \mathcal{A} \rightarrow \mathbb{C}, \\ A \mapsto E_z(A) \in \mathbb{C}, \end{aligned} \tag{35}$$

with

$$E_z(\mathbb{1}) = 1. \tag{36}$$

The existence of an expectation value functional has far reaching consequences

- Addition of observables and multiplication of observables with complex numbers can now be defined by postulating

$$E_z(\alpha A + \beta B) = \alpha E_z(A) + \beta E_z(B) \tag{37}$$

for all E_z. Axioms A1–A10 are fulfilled, A11 seems to be natural, less so A12.

- Being the mean value of a probability distribution, $E_z(A)$ has to obey reality and positivity conditions. The only evident way to achieve this is the introduction of a star-involution $A \to A^*$ with the properties S1–S3. (S2 has to hold, because A^*A has to be self-adjoint also if A and A^* do not commute.) Reality and positivity of E_z mean that Z1 and Z2 have to hold for all E_z.

- The set of all expectation value functionals will be convex. Pure states can be defined as in ordinary quantum theory.

The axioms B1–B5 are almost mandatory if one assumes that $\mathcal{A}$ can be topologized by a norm. The C^*-axiom C1 is least intuitive. Assuming C1 on top of the other axioms A, S, B, Z, ordinary quantum theory can be recovered. As mentioned above, only a detailed analysis of the concrete situation can decide which axioms are fulfilled.

5 Complementarity and entanglement in weak quantum theory: two applications

In this section, we outline two examples for the application of weak quantum theory. As mentioned in the preceding section, it should be possible to construct frameworks less restrictive than ordinary quantum theory but more restrictive than the weak version. Our first example, the complementarity of different types of dynamical descriptions of physical systems, addresses precisely such a situation. This example is particularly interesting since it can be presented in a fairly well formalized manner.

The second example, addressing transference and countertransference phenomena in psychology, will be discussed in an entirely qualitative and informal way. The basic complementarity in this example is that of conscious and unconscious processes, hence the relevant states and observables are mental, not material. It is likely that this example refers to the minimal set of axioms given in the preceding section.

There are additional possibilities to illustrate the applicability of weak quantum theory. An example which is intended to be worked out in detail elsewhere addresses a complementarity between the effects of placebo substances and “true” treatment substances in double-blind clinical trials. It is understood to refer to the effects of such substances in a purely pharmacological (physiological) sense, i.e., without considering possible psychological aspects.

5.1 Information Dynamics

Generalizing earlier work by Misra (1978) and Misra et al. (1979), an information theoretical description of chaotic systems (including K-systems) was found to provide a commutation relation between the Liouville operator L for such systems and a suitably defined information operator M (Atmanspacher and Scheingraber 1987). The definition of L is, as usually, given by

$$L\ \rho = i \frac{\partial}{\partial t}\ \rho \tag{38}$$

where L acts on distributions ρ which represent the states of a system in a usual probability space (not in a Hilbert space). The continuous spectrum of M derives from the time-dependent information $I(t)$ which can be gained by measuring particular properties of a system at time t in comparison with its predicted properties:

$$M\ \rho = I(t)\ \rho = (I(0) + Kt)\ \rho \tag{39}$$

K is the Kolmogorov-Sinai entropy, a statistical dynamical invariant of the system. It is experimentally available by Grassberger-Procaccia type algorithms (Grassberger and Procaccia 1983). $K > 0$ only for chaotic systems with intrinsically unstable dynamics. In an information theoretical interpretation (Shaw 1981), K characterizes the rate at which the system generates information along its unstable manifolds. Kt is the information generated by the system between t and $t = 0$. This means that the accuracy of a prediction decreases with increasing prediction time.

In simple cases, the commutator of L and M is just given by the rate of information generation, namely the Kolmogorov-Sinai entropy:

$$i[L, M] = K\mathbb{1} \tag{40}$$

The two operators commute precisely if the considered system does not generate information, i.e., if it is intrinsically stable. If $K > 0$, the dynamical descriptions due to L and M are different with respect to the prediction of a future state of the system. This is a consequence of the increasing uncertainty in predicting the state of a system as time proceeds. Whenever $K > 0$, the state $\rho(t)$ of a system cannot be predicted as accurate as initial conditions have been measured or otherwise fixed at $t = 0$.

The commutation relation of L and M resembles corresponding commutation relations in ordinary quantum theory, but there are differences. First of all, since K is explicitly system- and parameter-dependent (i.e. highly contextual), the "degree" of non-commutativity of L and M is not universally the same. This situation is at variance with conventional quantum mechanics with h as a universal commutator. Moreover, K is a statistical quantity specifying the average flow of information in chaotic systems, while h is a non-statistical constant of nature.

As a consequence of relation (40), L and M provide complementary modes of description. There are two basic features of this complementarity. (i) While L

refers to an ontic, completely deterministic description, M refers to an epistemic, coarse grained description of explicitly statistical nature (cf. Atmanspacher 2000). (ii) While a description in terms of L is time-reversal symmetric (reversible), this symmetry is broken by a description in terms of M, thus leading to irreversibility.

There is an interesting relation between (40) and another commutation relation between L and a time operator T introduced by Misra (1978) and Misra et al. (1979):

$$i[L, T] = \mathbb{1} \tag{41}$$

T is well-defined if $K > 0$. Since L, in addition to its role as an evolution operator as in (38), can also be interpreted as an energy difference due to $L\rho = [H, \rho]$, (41) indicates a complementarity between energy and time for chaotic systems. This suggests the idea of a temporal entanglement for such systems. This entanglement can be interpreted as a temporal nonlocality (Misra and Prigogine 1983) due to a coarse grained phase space; for a more detailed discussion see Atmanspacher (1997). It should be emphasized that this nonlocality is epistemic and must not be mixed up with the ontic nonlocality of ordinary quantum theory.

An interpretation of the commutation relation between L and M in terms of propositions leads to a lattice theoretical analysis. Analogous to the work of Birkhoff and von Neumann (1936) which pioneered the non-Boolean logic of quantum theory, such an analysis provides basic logical features of information processing systems. Following an idea by Krueger (1984), it was shown that the temporal evolution of information processing systems is governed by a non-Boolean logic (Atmanspacher 1991a). More precisely, the propositional lattice characterizing such a logic is complemented but not distributive, reflecting the complementarity of propositions from the perspective of a lattice theoretical formulation. The non-distributivity of this lattice, however, shows a subtle but important difference as compared with the non-distributivity due to ordinary quantum theory.

A fundamental feature of lattices as mathematical structures is the duality of their properties. Formally this means that each true proposition is transformed into another true propositon by exchanging the dual operations defined in lattice theory. It turns out that the subtle difference between the standard quantum theoretical non-distributivity and the non-distributivity due to information processing systems precisely accounts for this duality. While standard quantum theory provides non-distributivity relations of the form

$$\begin{aligned} & a > (a \wedge b) \vee (a \wedge b') \\ \wedge \quad & b > (b \wedge a) \vee (b \wedge a') \end{aligned} \tag{42}$$

(a' ist the complement of proposition a, b' is the complement of proposition b), information processing systems satisfy non-distributivity relations of the form:

$$\begin{aligned} & a < (a \vee b) \wedge (a \vee b') \\ \vee \quad & b < (b \vee a) \wedge (b \vee a') \end{aligned} \tag{43}$$

By contrast to (42), (43) requires only one of the two inequalities to be satisfied. A detailed analysis (Atmanspacher 1991a) shows that this is indeed crucial for the non-distributivity of information processing systems. It is therefore tempting to consider the logics of standard quantum systems and of information processing systems as dual aspects of one underlying non-distributive lattice (Atmanspacher 1991b).

5.2 Countertransference Phenomena

Freud (1992) was the first to observe that in the context of a therapeutic relationship strange interpersonal experiences can happen which he called transference and countertransference. Transference normally refers to the fact that the patient activates conflictual relationship themes from his past and enacts them in the context of the therapeutic relationship, transferring these past experiences into the presence and acting as if the therapist was his mother, father, brother or whoever he had the conflictual relationship with. Modern therapeutic theories postulate that a potentially helpful therapy will in fact activate such past experiences in the presence. By countertransference Freud originally meant that also within the therapist some potentially conflictual material can be activated by the patient, if the therapist is prone to the same problematic pattern as the patient.

In addition to the traditional and straigthforward meaning of transference and countertransference phenomena there is also a more subtle meaning which we will discuss in the following. It refers to a therapist's experience of inner states like emotions, ideas, thoughts, inner images, impulses, needs, phantasies, wishes, which are in fact "transferred" from the patient and reflect the inner state of the patient rather than that of the therapist. In clinical practice this countertransference phenomenon is used both diagnostically and interventionally. Diagnostically it can be a source of direct and intuitive information about the inner world of the patient. Thus, if a therapist experiences, in an otherwise calm atmosphere with a patient talking serenely about something pleasant which he or she has experienced, sudden throngs of aggression or wild phantasies of sexual abuse, then he might tentatively isolate these inner experiences as possibly belonging to the patient rather than to himself. He might then operate with the hypothesis that the calm story of the patient is just the surface, while underneath there might lie some material of a more dire nature.

Depending on the school of therapy the therapist belongs to, the nature of the problem, the state of the therapy, and the personality of the patient the therapist might choose to express this phantasy explicitly and offer it to the patient as an interpretative framework. Alternatively, he might feed back his own inner world, without any qualifications, as is often done in Gestalt therapy. Or he may keep it as an information which could direct later interventions. In any case, the hypothesis on which therapists often base their interventions is that the material they experience themselves under certain circumstances derives from the patient rather than from themselves. Although it is, strictly speaking, not possible to give clearcut rules as to when material is transferred or not, there are some practical rules of thumb. If material feels alien or strange to the therapist's state of mind, if it does not "fit" with the rest of the situation, then there is a good chance that the material is from the patient and not from the therapist himself. Therapeutic training within the depth psychological schools of therapy places a lot of emphasis and takes great pains and care to sharpen the inner awareness for the subtle changes which signal transference processes.

A comparable phenomenon is known from system-therapeutic settings, where family constellations are enacted in a group. In some schools the person working on their family will, with the help of the therapist, call other group members onto a stage or set place. These group members, the so called protagonists, are to take the place of the family members. Even family members who are long deceased or whom the patient does not know much about might be included in the family picture. It has been repeatedly observed that protagonists suddenly experience mental states which belong to the person exemplified, without the protagonist explicitly knowing about any corresponding details. For instance a protagonist representing a relative who has committed suicide, a fact unknown to all persons, might suddenly feel the impulse to leave the room. Or another protagonist, who represents someone who had a severe war injury might suddenly complain of strong pain, although nobody in the room, except perhaps the client, is aware of this fact. System theoretic therapists call this phenomenon "participatory" or "deputy" perception (Varga v. Kibed 1998).

There are reasons to assume that this is a phenomenon of the same type as in a classical countertransference situation. In both cases it is supposed that someone experiences mental states which do not pertain to himself but to someone else, e.g. the patient or some other person who is represented in the family setting. The phenomenon is well known as such, but for a lack of theoretical explanation has not found much interest, except for some practical purposes. We propose here to see this phenomenon as an example of entangled mental states, a very general situation addressed by weak quantum theory.

A crucial condition for successful therapeutic relationships is mutual openness. Ideally, both the therapist and the client communicate without withholding important or possibly important information. While in the psychoanalytic tradition this is a significant part of an explicit agreement, it certainly is also implictly true for most therapeutic alliances. But this openness is opposed by all those parts of inner material which are not available for conscious processing, either because they are subconscious and not known, or because they are not identified as having a particular flavor or relevance.

In order to apply the concepts of weak quantum theory to such situations, we consider the entire group of involved people as the system as a whole. The subsystems are the individual members of the group with particular emphasis on their mental (psychological) variables. The local preparation of "conscious awareness" can then be considered as complementary to a global preparation of material which is principally not available, because it is unconscious or irrelevant. While the latter corresponds to a global observable of the system as a whole (maybe referring to some kind of collective unconscious material), the former corresponds to local observables of subsystems, i.e. conscious contents of the mental system of individuals.

Material which is unconscious cannot be consciously known or even openly communicated, and contents which are consciously known or can be communicated openly cannot be unconscious. In this sense, the concepts of consciousness and unconscious are complementary. They are not only opposed to each other, but preclude each other and at the same time are both necessary for a complete picture of the overall mental system.

The process leading from unconscious material to (partly) conscious manifestations of that material may be conceived as a psychological analogue of the physical process of observation (cf. Jung 1971). In both types of processes, a global state is decomposed into a local state plus an environment, where the environment is assumed to include the measuring apparatus. In the psychological case, this means that a part of consciousness is the analogue of a "measuring tool" and another, emerging part of consciousness is the analogue of the physical subsystem emerging from the system as a whole.

In close analogy to the quantum situation, where measurement separates ontic (holistic) and epistemic (local) levels of description, the appearance of conscious contents as manifestations of unconscious material must be considered as a transformation between fundamentally different mental modalities. The unconscious mode is left (and maybe even changed) whenever a conscious content emerges out of it. Long ago, James (1950) perfectly paraphrased this situation by the impossibility to recognize what darkness is by switching the light on.

This difference between the two modalities becomes particularly interesting if the (unconscious) global state is in fact an entangled state. The entanglement can then refer to unconscious personal material or to the unconscious of collectives, resembling a specific realization of Jung's concept of the collective unconscious. In the case of individuals and their unconscious, the global system would correspond to some undifferentiated personal realm of the unconscious without local, separate categories, while elements of consciousness, such as mental categories, are local and separate. Particular mental categories (including the "I" or "self" as one of the most significant among them) are conceived to emerge by the transformation of unconscious material into consciously and empirically accessible categories.

In order to discuss transference and countertransference phenomena, it is necessary to address more than one individual. This makes it mandatory to consider the (collective) unconscious of a group, such as described in the examples above. If by some sort of "organizational closure" individuals establish a tightly bound system – a pair of lovers, a family, or another social group – then novel conscious contents can emerge at some particular part of the system (e.g., in one individual) as a result of a manifestation of unconscious material within the system as a whole. Again, it should be emphasized that unconscious material is not simply "made conscious" as it is; the emergence of conscious manifestations of unconscious material must be understood as a transition between fundamentally different mental modalities.

Conversely, if the binding is intense enough, personal unconscious material of one individual can become part of the collective unconscious of the system as a whole by some kind of composition (rather than decomposition) process. In this way the collective unconscious of the system as a whole becomes a "melting pot" of highly correlated individual contributions, to be formally described as an entangled state.

Although the mechanisms of decomposition and composition for such a scenario are far from being explored in detail, the basic framework of weak quantum theory offers an interesting perspective for what can happen in transference and counter-transference processes even beyond therapeutic applications. For instance, if in a marriage relationship one of the partners "experiences" something which is systemically unconscious, say the wish to separate, then this wish can manifest itself in the other partner's awareness as his or her own wish. The fatal aspect of such a phenomenon is that the corresponding material is mostly taken at face value rather than as a possible indicator of something originally belonging to another person.

6 Summary

The core content of this paper is the formulation of a weak version of quantum theory. It is motivated by the attempt to find a formal framework for addressing the concepts of complementarity and entanglement not only within the context of ordinary quantum physics, but also in more general contexts. The weak version of quantum theory is based on a minimal set of axioms. The basic structure of the resulting mathematical framework is that of a monoid.

Ordinary quantum theory can be recovered from this framework by additional axioms, restrictions, and specifications. For example, the weak version does not necessarily entail a Hilbert space representation or a probabilistic interpretation. The non-commutativity of observables is not necessarily quantified by Planck's constant. Bell-type inequalities cannot necessarily be formulated in weak quantum theory.

Among the many examples for complementary relations that can be found in the literature, two case studies were presented to demonstrate the applicability of weak quantum theory. They refer to (1) complementary types of dynamical descriptions of physical systems, and (2) the relation between conscious and unconscious processes in psychoanalytic and psychotherapeutic settings.

These examples show that there are different levels of generalization between weak quantum theory and ordinary quantum theory, depending on which restrictions are added to the minimal, weak framework. While example (2) is likely to need the full generalization of the weak version, only a few conditions of ordinary quantum theory are relaxed in example (1). These conditions are discussed to some detail.

The main benefit of weak quantum theory is its applicability beyond physics. This is possible since the generalized formal framework is conceptually decoupled from its application to ordinary quantum physics. Although "naive" analogies to the physical domain remain helpful, the wider scope allows a formally based approach to non-physical situations as well. Clearly there may be situations which are far from a formal description in terms of non-commutative operators as yet. On the other hand, there are concrete proposals, e.g. for cognitive processes (cf. Gernert 2000) toward corresponding descriptions which underline the potential usefulness of weak quantum theory.

Different future steps to explore this potential usefulness are conceivable. One of them is a more formal and detailed discussion of applications, which would go beyond the frame of the present paper. One example concerning placebo research was mentioned already. Another, much more general problem refers to the question how psychophysical relationships could be treated within weak quantum theory. Needless to say, it will be crucial to propose and carry out experiments demonstrating the full power of the approach.

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